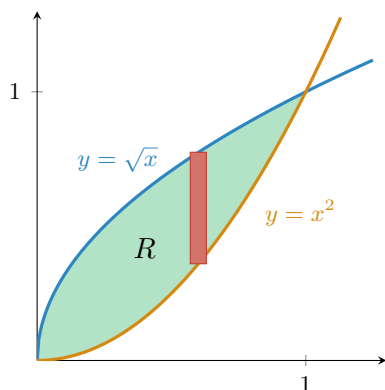


Area Between Curves and Volumes

AP Calculus AB | Unit 8 | area, known cross sections, discs and washers

One idea, used every time: **cut the region into thin slices, write what one slice contributes, integrate.** Throughout, R is the region between $y = \sqrt{x}$ and $y = x^2$.

1. Area between two curves



A vertical slice has height (top – bottom) and width dx :

$$A = \int_a^b (\text{top} - \text{bottom}) dx.$$

The limits are where the curves meet: $\sqrt{x} = x^2$ at $x = 0$ and $x = 1$.

$$A = \int_0^1 (\sqrt{x} - x^2) dx = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}.$$

Horizontal slices: $\int (\text{right} - \text{left}) dy$, with x in terms of y .

Curves cross inside the interval: split the integral there (top and bottom swap).

2. Volumes with known cross sections

R is the base; each slice perpendicular to the x -axis is a flat shape of area $A(x)$.

Known cross sections

$$V = \int_a^b A(x) dx$$

Here the side of each shape is $s = \text{top} - \text{bottom} = \sqrt{x} - x^2$. For **squares**, $A = s^2$:

$$V = \int_0^1 (\sqrt{x} - x^2)^2 dx = \frac{9}{70}.$$

square	s^2	rectangle of height h	$s \cdot h$
semicircle (diameter s)	$\frac{\pi}{8}s^2$	equilateral triangle	$\frac{\sqrt{3}}{4}s^2$
isosceles right triangle (leg s)	$\frac{1}{2}s^2$		

3. Discs and washers

Revolving R about a horizontal line makes every slice sweep out a ring (a washer).

Washer method

$$V = \pi \int_a^b (R^2 - r^2) dx$$

R = outer radius, r = inner radius, both measured **from the axis of rotation**. If the region touches the axis, $r = 0$: the disc method.

About the x -axis: $R = \sqrt{x}$, $r = x^2$:

$$V = \pi \int_0^1 (x - x^4) dx = \pi \left(\frac{1}{2} - \frac{1}{5} \right) = \frac{3\pi}{10}.$$

About the line $y = -1$: every distance is one unit longer, $R = \sqrt{x} + 1$, $r = x^2 + 1$:

$$V = \pi \int_0^1 ((\sqrt{x} + 1)^2 - (x^2 + 1)^2) dx = \frac{29\pi}{30}.$$

Where students lose points

1. $(R - r)^2$ instead of $R^2 - r^2$.
2. Forgetting π in a volume of revolution.
3. Radii not measured from the axis of rotation.
4. Bottom – top, or wrong limits: find the intersections.
5. Cross sections: using the washer formula, or the wrong area formula.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

sad-academy.com