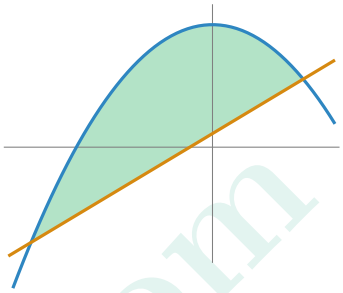
Area Between Curves

AP Calculus AB | Unit 8 | one slice, then add them up

1. Top minus bottom

Cut the region into thin vertical slices. A slice at x has height (top − bottom) and width dx; adding the slices gives an integral.



Vertical slices

$$A = \int_a^b (\text{top} - \text{bottom})\, dx,$$

with a and b where the curves meet.

Example. $y = 9 - x^2$ and $y = 2x + 1$. Setting them equal: $x^2 + 2x - 8 = 0$, so $x = -4$ and $x = 2$, and

$$A = \int_{-4}^{2} (9 - x^2 - (2x + 1))\, dx = \left[8x - x^2 - \tfrac{x^3}{3}\right]_{-4}^{2} = 36.$$

2. When the curves cross

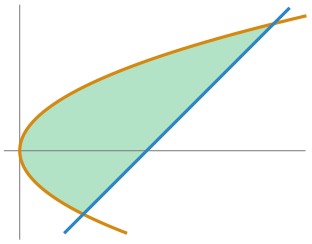
$y = x^3 - 3x$ and $y = x$ meet at $x = -2, 0, 2$. On $[-2, 0]$ the cubic is on top; on $[0, 2]$ the line is. A single integral $\int_{-2}^{2} (x - (x^3 - 3x))\, dx = 0$ lets the pieces cancel. Split at every crossing:

$$A = \int_{-2}^{0} ((x^3 - 3x) - x)\, dx + \int_{0}^{2} (x - (x^3 - 3x))\, dx = 4 + 4 = 8,$$

or, with a calculator, $A = \int_{-2}^{2} |\, x - (x^3 - 3x)\,|\, dx$.

3. Slicing sideways: right minus left

When the region is bounded by curves written as x in terms of y, slice horizontally:



Horizontal slices

$$A = \int_c^d (\text{right} - \text{left})\, dy,$$

with c and d the y-values where the curves meet.

Example. $x = \frac{y^2}{2}$ and $x = y + 4$ meet at $y = -2$ and $y = 4$, so $A = \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2}\right) dy = 18$. With vertical slices the lower boundary would change at $x = 2$, and it would take two integrals.

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Where students lose points

1. Bottom minus top: a negative "area" means the order is reversed.
2. Guessing the limits instead of setting the curves equal.
3. Not splitting where the curves cross, so the pieces cancel.
4. With horizontal slices: using top − bottom, or not writing both curves as x in terms of y.
5. Mixing variables: an integrand in x with limits in y.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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