

Average Value of a Function

AP Calculus AB | Unit 8 | the formula, the picture, average value vs average rate of change

1. From a sum to an integral

Take n equally spaced values of f on $[a, b]$ and average them. Since $\Delta x = \frac{b-a}{n}$, we have $\frac{1}{n} = \frac{\Delta x}{b-a}$, so

$$\frac{f(x_1) + \cdots + f(x_n)}{n} = \frac{1}{b-a} \sum_{i=1}^n f(x_i) \Delta x \xrightarrow{n \rightarrow \infty} \frac{1}{b-a} \int_a^b f(x) dx.$$

Average value of f on $[a, b]$

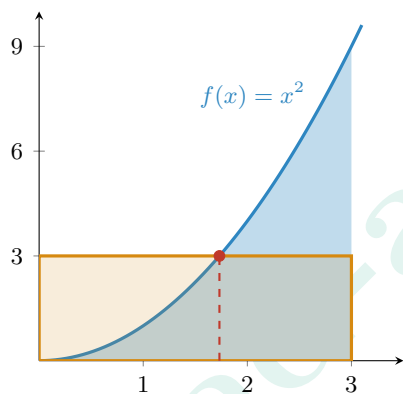
$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx$$

The integral divided by the **length of the interval**. Units: the same as f .

Example. Temperature $T(t) = 20 + 6 \sin\left(\frac{\pi t}{12}\right)$ °C for $0 \leq t \leq 12$ hours:

$$T_{\text{avg}} = \frac{1}{12-0} \int_0^{12} \left(20 + 6 \sin \frac{\pi t}{12}\right) dt = 20 + \frac{12}{\pi} \approx 23.82 \text{ °C}.$$

2. What it looks like



For $f(x) = x^2$ on $[0, 3]$: $\int_0^3 x^2 dx = 9$, so $f_{\text{avg}} = \frac{1}{3-0} \cdot 9 = 3$.

The average value is the **height of the rectangle** on the same interval that has the **same area** as the region under the curve ($3 \cdot 3 = 9$).

A continuous function takes its average value somewhere on the interval: $f(c) = 3$ gives $c = \sqrt{3} \approx 1.73$.

Another one. $\sin x$ on $[0, \pi]$: $\frac{1}{\pi-0} \int_0^\pi \sin x dx = \frac{2}{\pi} \approx 0.64$.

3. Average value vs average rate of change

	formula	units
Average value of f	$\frac{1}{b-a} \int_a^b f(x) dx$	same as f
Average rate of change of f	$\frac{f(b) - f(a)}{b-a}$	f per unit of x

The link (Fundamental Theorem): the average value of f' equals the average rate of change of f ,

$$\frac{1}{b-a} \int_a^b f'(x) dx = \frac{f(b) - f(a)}{b-a}.$$

In context, given a velocity $v(t)$: **average velocity** is the average *value* of v (an integral: displacement $\div$ time); **average acceleration** is the average *rate of change* of v (a subtraction).

Where students lose points

1. Forgetting the $\frac{1}{b-a}$: giving the integral as the answer.
2. Using $\frac{f(b)-f(a)}{b-a}$: that is the average rate of change.
3. Averaging the endpoints, $\frac{f(a)+f(b)}{2}$.
4. Dividing by b instead of by $b-a$.
5. Wrong or missing units: the average has the units of f .

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

sad-academy.com