Average Value of a Function

AP Calculus AB | Unit 8 | the formula, the picture, graphs and tables, average value vs average rate of change

1. From a sum to an integral

Take n equally spaced values of f on $[a,b]$ and average them. Since $\Delta x = \dfrac{b-a}{n}$, we have $\dfrac{1}{n} = \dfrac{\Delta x}{b-a}$, so

$$\frac{f(x_1)+\cdots+f(x_n)}{n} = \frac{1}{b-a}\sum_{i=1}^{n} f(x_i)\,\Delta x \xrightarrow{n\to\infty} \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Average value of f on $[a,b]$

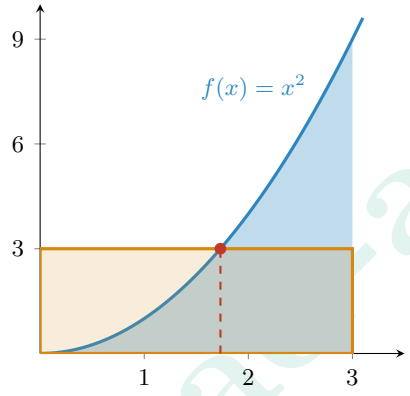
$$f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

The integral divided by the **length of the interval**. Units: the same as f.

Example. Temperature $T(t) = 20 + 6\sin\left(\frac{\pi t}{12}\right)$ °C for $0 \le t \le 12$ hours:

$$T_{\text{avg}} = \frac{1}{12-0}\int_0^{12}\left(20 + 6\sin\tfrac{\pi t}{12}\right)dt = 20 + \frac{12}{\pi} \approx 23.82\ ^\circ\text{C}.$$

2. What it looks like



For $f(x) = x^2$ on $[0,3]$: $\int_0^3 x^2\,dx = 9$, so $f_{\text{avg}} = \dfrac{1}{3-0}\cdot 9 = 3$.

The average value is the **height of the rectangle** on the same interval that has the **same area** as the region under the curve ($3 \cdot 3 = 9$).

A continuous function takes its average value somewhere on the interval: $f(c) = 3$ gives $c = \sqrt{3} \approx 1.73$.

Another one. $\sin x$ on $[0,\pi]$: $\dfrac{1}{\pi-0}\displaystyle\int_0^{\pi}\sin x\,dx = \frac{2}{\pi} \approx 0.64$.

3. Without a formula

A line. The region under a linear f is a trapezoid, so its area is $\frac{f(a)+f(b)}{2}\cdot(b-a)$ and

$$f_{\text{avg}} = \frac{f(a)+f(b)}{2} \qquad \text{(only when } f \text{ is linear).}$$

$f(x) = 3x+1$ on $[1,5]$: $\displaystyle\int_1^5 (3x+1)\,dx = \frac{4+16}{2}\cdot 4 = 40$, $f_{\text{avg}} = 10$. For a curve the shortcut fails: x^2 on $[0,3]$ gives $\frac{0+9}{2} = 4.5$, but $f_{\text{avg}} = 3$.

A graph made of shapes. Find the integral as a *net signed* area: regions below the x-axis count as negative. g = semicircle of radius 2 on $[0,4]$, then a segment from $(4,0)$ to $(8,-2)$:

$$\int_0^8 g = \tfrac{1}{2}\pi(2)^2 - \tfrac{1}{2}\cdot 4\cdot 2 = 2\pi - 4, \qquad g_{\text{avg}} = \frac{2\pi-4}{8} \approx 0.29.$$

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Comparing with one value. Find the average first, then compare. A valley from $(0,5)$ to $(3,1)$ to $(6,5)$: area $= 9+9 = 18$, $f_{\text{avg}} = 3 > f(3) = 1$. For a peak the comparison flips.

4. Pieces, tables, shorter intervals

Pieces. Turn each average back into an integral (average $\times$ length), add, divide by the whole length. Average 6 on $[0,4]$ and 3 on $[4,6]$: $f_{\text{avg}} = \frac{6\cdot 4+3\cdot 2}{6} = 5$, **not** $\frac{6+3}{2} = 4.5$.

A table. Approximate the integral with trapezoids (unequal widths are fine), then divide by the total length:

t (hours)	0	2	5	6
$T(t)$ (°F)	60	64	70	62

$$\int_0^6 T\,dt \approx \tfrac{60+64}{2}\cdot 2 + \tfrac{64+70}{2}\cdot 3 + \tfrac{70+62}{2}\cdot 1 = 391, \qquad T_{\text{avg}} \approx \frac{391}{6} \approx 65.2\ ^\circ\text{F}.$$

A shorter interval. Ask which values you throw away. f decreasing on $[0,8]$: the right end holds the smallest values, so the average on $[0,4]$ is *larger* than on $[0,8]$ (for $8 - \frac{x^2}{8}$: ≈ 7.3 vs ≈ 5.3). For an increasing f it is smaller.

5. Average value vs average rate of change

	formula	units
Average value of f	$\dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$	same as f
Average rate of change of f	$\dfrac{f(b)-f(a)}{b-a}$	f per unit of x

The link (Fundamental Theorem): the average value of f' equals the average rate of change of f,

$$\frac{1}{b-a}\int_a^b f'(x)\,dx = \frac{f(b)-f(a)}{b-a}.$$

Given an antiderivative F of f: $f_{\text{avg}} = \dfrac{F(b)-F(a)}{b-a}$. In context, given a velocity $v(t)$: **average velocity** is the average *value* of v (an integral: displacement $\div$ time); **average acceleration** is the average *rate of change* of v (a subtraction).

Where students lose points

1. Forgetting the $\frac{1}{b-a}$: giving the integral as the answer.
2. Using $\frac{f(b)-f(a)}{b-a}$: that is the average rate of change.
3. Averaging the endpoints, $\frac{f(a)+f(b)}{2}$, when f is not linear.
4. Dividing by b instead of by $b-a$.
5. Averaging two averages instead of weighting each by its length.
6. Wrong or missing units: the average has the units of f.

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