

The Central Limit Theorem

AP Statistics | Unit 5 | sampling distributions, CLT, unbiased estimators

1. Parameter vs statistic

Parameter	describes the population	μ, σ, p	fixed, usually unknown
Statistic	computed from a sample	$\bar{x}, s, \hat{p}$	known, varies from sample to sample

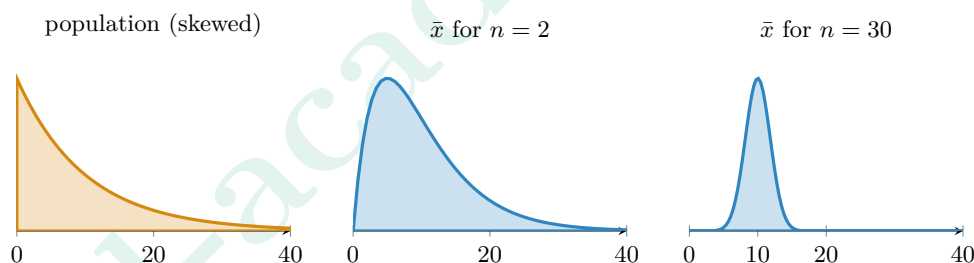
2. What a sampling distribution is

Three different distributions:

1. the **population**: every individual;
2. **one sample**: the n individuals you measured (it looks roughly like the population);
3. the **sampling distribution** of a statistic: the values of the statistic over *all possible samples* of the same size. Each “dot” is a whole sample.

The sampling distribution tells you how much the statistic varies from sample to sample.

3. The Central Limit Theorem



Central Limit Theorem

When the sample size n is **large**, the sampling distribution of the sample mean $\bar{x}$ is **approximately normal**, whatever the shape of the population.

Rule of thumb: $n \geq 30$. If the population itself is normal, $\bar{x}$ is normal for *any* n .

As n grows, the centre stays at μ and the spread shrinks (it is $\sigma/\sqrt{n}$; see the next guides). With a small n from a skewed population, the sampling distribution is still skewed.

4. Biased and unbiased estimators

Unbiased estimator

A statistic is an **unbiased estimator** of a parameter if the *mean* of its sampling distribution equals the parameter.

- $\bar{x}$ is an unbiased estimator of μ ; $\hat{p}$ is an unbiased estimator of p .
- Biased example: tickets are numbered 1 to 100; draw 5 and use the largest as an estimate of the top number. It can never exceed 100 and averages about 84: it systematically underestimates.

Bias is about the *centre* of the sampling distribution being in the wrong place; a larger sample does not fix it. **Variability** is about its *spread*; a larger sample reduces it.

Where students lose points

1. Confusing the distribution of a sample with the sampling distribution of a statistic.
2. “With a large n the data become normal.” It is the sampling distribution of $\bar{x}$ that becomes normal.
3. Using the CLT with a small n and a skewed population.
4. “A larger sample reduces bias.” It reduces variability.
5. Mixing up μ and $\bar{x}$, or p and $\hat{p}$.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you’ll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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