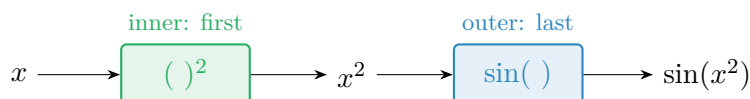


The Chain Rule

AP Calculus AB | Unit 3 | compositions, layers, choosing a rule

1. Functions inside functions

In $\sin(x^2)$, x is squared first (the **inner** function), then the result goes into sine (the **outer** function): a composition $f(g(x))$.



function	outer	inner
$(3x + 1)^5$	$()^5$	$3x + 1$
e^{3x}	$e^{()}$	$3x$
$\sqrt{x^2 + 1}$	$\sqrt{()}$	$x^2 + 1$
$\sin^2 x = (\sin x)^2$	$()^2$	$\sin x$

2. The chain rule

Rates multiply: if u changes 3 times as fast as x and y changes 2 times as fast as u , then y changes $2 \cdot 3 = 6$ times as fast as x .

Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad \Longleftrightarrow \quad \frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

Derivative of the *outside* with the inside left alone, times the derivative of the *inside*.

$$\frac{d}{dx}[(x^2 + 1)^{10}] = 10(x^2 + 1)^9 \cdot 2x = 20x(x^2 + 1)^9, \quad \frac{d}{dx}[\sin(x^2)] = \cos(x^2) \cdot 2x,$$

$$\frac{d}{dx}[e^{3x}] = 3e^{3x}, \quad \frac{d}{dx}[(3x + 1)^5] = 5(3x + 1)^4 \cdot 3 = 15(3x + 1)^4.$$

3. More than one layer

One factor per layer, from the outside in:

$$\frac{d}{dx}[\sin^3(4x)] = 3[\sin(4x)]^2 \cdot \cos(4x) \cdot 4 = 12\sin^2(4x)\cos(4x).$$

A chain inside a product: product rule on the outside, chain rule on the piece that needs it.

$$\frac{d}{dx}[x^2 e^{3x}] = 2x e^{3x} + x^2 \cdot 3e^{3x}.$$

Which rule? Ask what the *last* operation is

$x^3 + \sin x$	a sum	differentiate term by term
$x^3 \sin x$	a product	product rule
$\frac{\sin x}{x^3}$	a quotient	quotient rule
$\sin(x^3)$	a function of a function	chain rule

Then ask the same question again for each piece.

4. How the exam asks it

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	-4	3	2
3	7	5	1	6

If $h(x) = f(g(x))$, then $h'(1) = f'(g(1)) \cdot g'(1) = f'(3) \cdot 2 = 5 \cdot 2 = 10$. Work from the inside: $g(1) = 3$ tells you *which row* to read f' from. The trap answer is $f'(1) \cdot g'(1) = -8$.

Where students lose points

1. Forgetting to multiply by the derivative of the inside.
2. Changing the inside: writing $f'(g'(x))$.
3. Table problems: $f'(1)$ instead of $f'(g(1))$.
4. Stopping after one layer when there are three.
5. Misreading $\sin^2 x$ as $\sin(x^2)$.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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