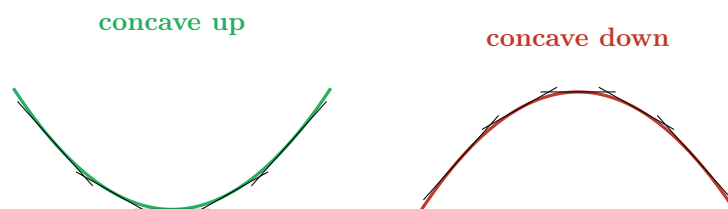


Concavity & Inflection Points

AP Calculus AB | Unit 5 | the second derivative and the bend of a graph

1. What concavity means



- **Concave up:** the slopes increase from left to right (f' is increasing); tangent lines lie below the curve.
- **Concave down:** the slopes decrease (f' is decreasing); tangent lines lie above the curve.

Concavity is not about f going up or down: two increasing functions can bend in opposite ways.

2. The second derivative decides

The rule

$f''(x) > 0$ on an interval $\Rightarrow f$ is **concave up** there.

$f''(x) < 0$ on an interval $\Rightarrow f$ is **concave down** there.

$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 2$ has $f''(x) = 2x - 2$: negative for $x < 1$ (concave down), positive for $x > 1$ (concave up).

3. Points of inflection

Definition

A **point of inflection** is a point where the concavity changes: f'' **changes sign** (and f is continuous there).

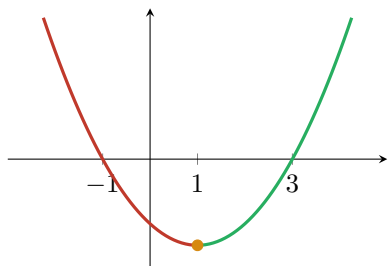
$f''(c) = 0$ is *not* enough. For x^4 , $f''(0) = 0$ but $f'' = 12x^2 \geq 0$: no sign change, no inflection point.

For the function above the point of inflection is $(1, -\frac{5}{3})$.

Example. $g''(x) = (x - 2)^2(x + 1)$ is zero at $x = -1$ and $x = 2$. Its sign is $-, +, +$: it changes at -1 but not at 2 . So g has exactly one point of inflection, at $x = -1$.

4. Reading the graph of f'

the graph of $y = f'(x)$



- f' **decreasing** $\Rightarrow f$ concave down.
- f' **increasing** $\Rightarrow f$ concave up.
- f' has a relative max or min $\Rightarrow f$ has a point of inflection.

Two different questions on the same graph of f' : *above or below the axis* tells you whether f is increasing or decreasing; *rising or falling* tells you the concavity of f .

Justifications. “ f is concave up on $(1, \infty)$ because $f''(x) > 0$ there” (or “because f' is increasing there”). “ f has a point of inflection at $x = 1$ because f'' changes sign there.”

Where students lose points

1. “ $f''(c) = 0$, so it is a point of inflection.”
2. Confusing concave up with increasing.
3. On a graph of f' : using the wrong feature for the question.
4. Justifying an inflection point without “changes sign”.
5. Giving x when the question asks for the *point*.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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