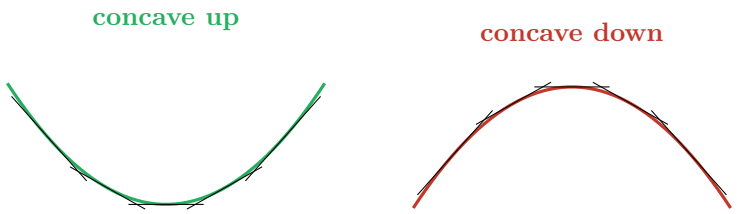
Concavity & Inflection Points

AP Calculus AB | Unit 5 | the second derivative and the bend of a graph

1. What concavity means



- **Concave up**: the slopes increase from left to right (f' is increasing); tangent lines lie below the curve.
- **Concave down**: the slopes decrease (f' is decreasing); tangent lines lie above the curve.

Concavity is not about f going up or down: two increasing functions can bend in opposite ways.

2. The second derivative decides

The rule

$f''(x) > 0$ on an interval $\Rightarrow$ f is **concave up** there.
$f''(x) < 0$ on an interval $\Rightarrow$ f is **concave down** there.

$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 2$ has $f''(x) = 2x - 2$: negative for $x < 1$ (concave down), positive for $x > 1$ (concave up).

3. Points of inflection

Definition

A **point of inflection** is a point where the concavity changes: f'' **changes sign** (and f is continuous there).

$f''(c) = 0$ is *not* enough. For x^4, $f''(0) = 0$ but $f'' = 12x^2 \geq 0$: no sign change, no inflection point.

For the function above the point of inflection is $\left(1, -\frac{5}{3}\right)$.

Example. $g''(x) = (x-2)^2(x+1)$ is zero at $x = -1$ and $x = 2$. Its sign is $-$, $+$, $+$: it changes at -1 but not at 2. So g has exactly one point of inflection, at $x = -1$.

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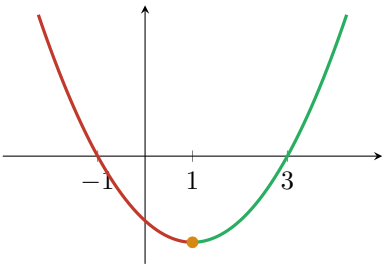
4. Reading the graph of f'

the graph of $y = f'(x)$



- f' **decreasing** ⇒ f concave down.
- f' **increasing** ⇒ f concave up.
- f' has a relative max or min ⇒ f has a point of inflection.

Two different questions on the same graph of f': *above or below the axis* tells you whether f is increasing or decreasing; *rising or falling* tells you the concavity of f.

Justifications. *"f is concave up on $(1, \infty)$ because $f''(x) > 0$ there"* (or *"because f' is increasing there"*). *"f has a point of inflection at $x = 1$ because f'' changes sign there."*

5. Tables, secants, and the domain

A table of f'. With $f'(1) = 2$, $f'(3) = 5$, $f'(5) = 6$, $f'(7) = 3$, the average rates of change of f' are 1.5, 0.5, −1.5; they estimate f''. So f appears concave up on $(1, 5)$ and concave down on $(5, 7)$.

f' given as an integral. If $f'(x) = \int_0^x k(t)\,dt$, then $f''(x) = k(x)$: f is concave up where $k > 0$, concave down where $k < 0$, and has a point of inflection where k changes sign. (For $k(t) = 10 - 2t$: up on $(0, 5)$, down after 5.)

Secant lines. If $f'' > 0$ on (a, b), the graph lies on or below the secant line from $(a, f(a))$ to $(b, f(b))$ and above its tangent lines; concave down is the reverse.

The Mean Value Theorem twice. f twice differentiable, $f(0) = 4$, $f(3) = 7$, $f(6) = 13$: the MVT gives $f'(c_1) = 1$ in $(0, 3)$ and $f'(c_2) = 2$ in $(3, 6)$; applied to f' it gives $f''(c) = \frac{2-1}{c_2-c_1} > 0$ for some c in $(0, 6)$.

The domain. $g(x) = \frac{1}{x-2}$ has $g''(x) = \frac{2}{(x-2)^3}$: concave down for $x < 2$, up for $x > 2$, but $x = 2$ is not in the domain, so it is *not* a point of inflection. Give intervals only inside the stated domain.

Where students lose points

1. "$f''(c) = 0$, so it is a point of inflection."
2. Confusing concave up with increasing.
3. On a graph of f': using the wrong feature for the question.
4. Justifying an inflection point without "changes sign".
5. Giving x when the question asks for the *point*.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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