Conditional Probability & Tree Diagrams

AP Statistics | Unit 2 | conditional probability, multiplication rule, tree diagrams

1. Given: shrink the world

A gym asks its 400 members about their membership and whether they take group classes.

	classes	no classes	total
monthly	48	112	160
annual	120	120	240
total	168	232	400

$P(\text{classes}) = \frac{168}{400} = 0.42$. Told that the member is annual, only the annual row is left: $P(\text{classes} \mid \text{annual}) = \frac{120}{240} = 0.50$.

Conditional probability

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \qquad \text{e.g.} \qquad \frac{120/400}{240/400} = \frac{0.30}{0.60} = 0.50$$

"|" means *given*. The given event is the denominator.

$P(A \mid B) \neq P(B \mid A)$: $P(\text{annual} \mid \text{classes}) = \frac{120}{168} \approx 0.71$, not 0.50.

2. The general multiplication rule

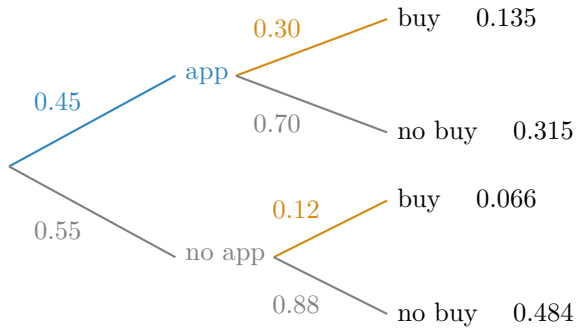
Multiplication rule

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

First A happens, then B happens under that condition.

- 45% of a store's customers use its app, and 30% of app users buy something: $P(\text{app and buy}) = 0.45 \times 0.30 = 0.135$.
- Two cards without replacement, both hearts: $\frac{13}{52} \cdot \frac{12}{51} \approx 0.059$. The second fraction is conditional: one heart is already gone.

3. Tree diagrams



- Second-stage branches are **conditional** on the first branch; each pair adds to 1.
- **Multiply along** a path: $0.45 \times 0.30 = 0.135$. All paths add to 1.
- **Add the paths** you want: $P(\text{buy}) = 0.135 + 0.066 = 0.201$.

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4. Flipping the condition

The tree is built app → buy. To find $P(\text{app} \mid \text{buy})$, use the formula:

$$P(\text{app} \mid \text{buy}) = \frac{P(\text{app and buy})}{P(\text{buy})} = \frac{0.135}{0.201} \approx 0.67.$$

Only 45% of customers use the app, but about 67% of buyers do.

A positive test

A disease affects 1% of people; a test catches 90% of people who have it and falsely flags 5% of healthy people. Out of 10,000 people: 100 are sick and 90 of them test positive; 9,900 are healthy and 495 of them test positive. So

$$P(\text{sick} \mid +) = \frac{90}{90 + 495} = \frac{90}{585} \approx 0.15.$$

The disease is rare, so false positives from the many healthy people outnumber the true positives. The 90% is $P(+ \mid \text{sick})$, not $P(\text{sick} \mid +)$.

5. More ways to condition

- **Equally likely outcomes**: the condition shrinks the list. Two dice, given at least one 6 (11 outcomes): $P(\text{sum} \geq 10) = \frac{5}{11}$, not $\frac{5}{36}$. Choose 2 of 5 students (3 girls, 2 boys): given at least one girl (9 of the 10 pairs), $P(\text{both girls}) = \frac{3}{9} = \frac{1}{3}$.
- **Complement inside a condition**: $P(\text{not } A \mid B) = 1 - P(A \mid B)$.
- **Find $P(A$ and $B)$ first**: $P(\text{lift}) = 0.64$, $P(\text{class}) = 0.42$, $P(\text{lift or class}) = 0.82$ give $P(\text{both}) = 0.64 + 0.42 - 0.82 = 0.24$, so $P(\text{lift} \mid \text{class}) = \frac{0.24}{0.42} \approx 0.57$. If the given event is a combination (brand A *or* brand B), its whole probability or count is the denominator.
- **Independence**: if $P(A \mid B) = P(A \mid \text{not } B)$, then B tells nothing about A: independent, and $P(A)$ equals that common value. If they differ, not independent.
- **Law of total probability** (the tree as one formula): $P(A) = P(A \mid B)P(B) + P(A \mid \text{not } B)P(\text{not } B)$. Pass exam 2 with 0.85 after passing exam 1 (probability 0.6) and 0.40 after failing: $0.85(0.6) + 0.40(0.4) = 0.67$.

Where students lose points

1. Dividing by the grand total: in a conditional probability the given group is the denominator.
2. Confusing $P(A \mid B)$ with $P(B \mid A)$, like a test's accuracy with the chance of disease.
3. Unconditional probabilities on the second-stage branches of a tree.
4. Adding along a path instead of multiplying (multiply along, add across).
5. Forgetting a path when finding $P(B)$: a customer can buy with the app or without it.
6. Keeping the whole sample space after a condition: count only the outcomes it leaves.

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