

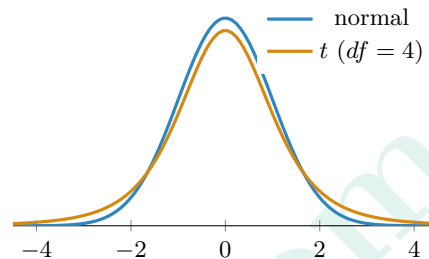
# Confidence Interval for a Mean

AP Statistics | Unit 7 | the  $t$  distribution,  $t^*$ , conditions, interpretation

## 1. From $z$ to $t$

The standard deviation of  $\bar{x}$  is  $\sigma/\sqrt{n}$ , but the population  $\sigma$  is never known. Replace it with the sample standard deviation  $s$ : the **standard error** of the mean is  $s/\sqrt{n}$ .

Because  $s$  is itself an estimate, there is extra uncertainty, described by the  **$t$  distribution**: bell-shaped and centred at 0 like the normal curve, but with *heavier tails*. There is one  $t$  curve for each number of **degrees of freedom**,  $df = n - 1$ ; as  $df$  grows it approaches the normal curve.



Heavier tails mean you must go farther out to capture 95%, so  $t^*$  is always larger than  $z^* = 1.96$ .

## 2. The one-sample $t$ interval

### One-sample $t$ interval for a mean

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}} \quad \text{with } df = n - 1$$

| $df$          | 4     | 14    | 29    | $\infty$ ( $z$ ) |
|---------------|-------|-------|-------|------------------|
| $t^*$ for 95% | 2.776 | 2.145 | 2.045 | 1.960            |

( $t^*$  comes from a table or from `invT` on the calculator.)

A random sample of 15 phones has mean battery life  $\bar{x} = 8.2$  h and  $s = 1.5$  h.  $df = 14$ ,  $t^* = 2.145$ ,  
 $\frac{s}{\sqrt{n}} = \frac{1.5}{\sqrt{15}} \approx 0.387$ .

$$8.2 \pm 2.145(0.387) = 8.2 \pm 0.83 \Rightarrow (7.37, 9.03) \text{ hours.}$$

## 3. The conditions

- **Random**: a random sample or a randomized experiment.
- **10%**:  $n$  is at most 10% of the population.
- **Normal / Large Sample**: the population is normal, or  $n \geq 30$ , or ( $n < 30$ ) a graph of the sample data shows no strong skewness and no outliers.

With a small sample, **sketch the graph** (dotplot, boxplot or histogram) and say what you see. “ $n < 30$  so we assume normal” earns nothing.

## 4. Interpreting, and what changes the width

*We are 95% confident that the interval from 7.37 to 9.03 hours captures the true mean battery life of all phones of this model.*

It is a statement about the **mean**, not about individuals: it does *not* say that 95% of phones last between 7.37 and 9.03 hours.

The interval is narrower with a larger  $n$ , a lower confidence level, or a smaller  $s$ . Using  $z^* = 1.96$  by mistake gives  $\pm 0.76$  instead of  $\pm 0.83$ : too narrow.

### Where students lose points

1. Using  $z^*$  for a mean instead of  $t^*$ .
2.  $df = n$  instead of  $df = n - 1$ .
3. Small sample: no graph, or " $n < 30$  so assume normal".
4. Interpreting the interval as about individuals.
5. Not naming the procedure: one-sample  $t$  interval for a mean.

### Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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