

Confidence Interval for a Proportion

AP Statistics | Unit 6 | formula, conditions, interpretation, sample size

1. Estimate $\pm$ margin of error

A sample proportion $\hat{p}$ is a **point estimate** of the unknown population proportion p . Because $\hat{p}$ varies from sample to sample, we report a range of plausible values for p :

point estimate $\pm$ margin of error.

2. Building the interval

One-sample z interval for a proportion

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

z^* is the **critical value**; the square root is the **standard error** of $\hat{p}$ (the standard deviation formula with $\hat{p}$ in place of the unknown p).

confidence level	90%	95%	99%
z^*	1.645	1.960	2.576

A random sample of 1000 voters; 520 support the candidate. $\hat{p} = 0.52$.

$$\sqrt{\frac{(0.52)(0.48)}{1000}} \approx 0.0158, \quad 1.96 \times 0.0158 \approx 0.031, \quad 0.52 \pm 0.031 \Rightarrow (0.489, 0.551).$$

3. The three conditions

- **Random:** the data come from a random sample.
- **10%:** n is at most 10% of the population (1000 is far less than 10% of all voters).
- **Large Counts:** $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ ($520 \geq 10$ and $480 \geq 10$).

Check each one with the actual numbers, and name the procedure.

4. Interpretation

Two sentences to know

The interval. We are 95% confident that the interval from 0.489 to 0.551 captures the true proportion of all voters who support the candidate.

The confidence level. If we took many random samples of this size and built an interval from each, about 95% of those intervals would capture the true proportion.

The level describes the *method*. It is **not** “a 95% probability that p is in this interval”: p is fixed, and one particular interval either contains it or does not. (Here the interval includes values below 0.50, so the poll does not show that the candidate is ahead.)

5. Width and sample size

- Higher confidence $\Rightarrow$ **wider**: at 99% the margin grows from 0.031 to 0.041.
- Larger sample $\Rightarrow$ **narrower**: with $n = 4000$ the margin is 0.0155. Four times the sample halves the margin.

Sample size for a given margin of error. Solve $z^* \sqrt{\frac{p^*(1-p^*)}{n}} \leq ME$ for n , using $p^* = 0.5$ when there is no prior estimate. For $ME = 0.02$ at 95%:

$$n \geq \left(\frac{1.96}{0.02}\right)^2 (0.5)(0.5) = 2401 \quad (\text{always round up}).$$

Where students lose points

1. “There is a 95% probability that p is in the interval.”
2. Interpreting the interval as about the sample, or about individuals.
3. Naming the conditions without checking them with numbers.
4. The wrong critical value ($z^* = 2$ instead of 1.96).
5. Rounding a sample size down.

Now practise it

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