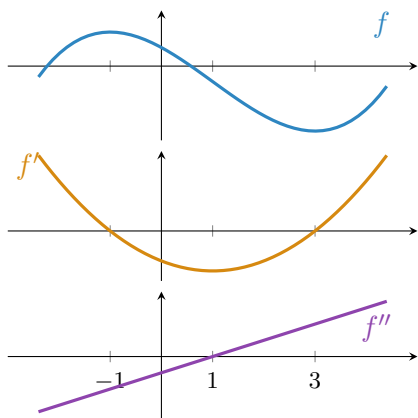


Connecting f , f' and f''

AP Calculus AB | Unit 5 | matching graphs, describing f from f' , optimization

1. Three graphs, lined up



- At $x = -1$ and $x = 3$, f has a max and a min; directly below, f' crosses zero.
- At $x = 1$, f has a point of inflection; f' has its minimum there, and f'' crosses zero.
- Where f is increasing, f' is above its axis. Where f is concave up, f'' is above its axis.

Each graph tells the slope story of the one above it.

2. The dictionary

f	f'	f''
increasing	positive	
decreasing	negative	
relative max / min	changes sign	
concave up	increasing	positive
concave down	decreasing	negative
point of inflection	relative max / min	changes sign

3. Matching unlabeled graphs

The ten-second method

Find where one graph has a maximum or minimum. Its derivative must be **zero** at exactly that x . Build the chain $f \rightarrow f' \rightarrow f''$ from there instead of guessing by shape.

4. Describing f from the graph of f'

Read the graph of f' twice.

- **Above or below the axis** (sign of f'): where f is increasing or decreasing, and its relative extrema.
- **Rising or falling** (is f' increasing?): where f is concave up or down, and its points of inflection.

Example. At a point where f' is positive and decreasing, f is increasing (because $f' > 0$) and concave down (because f' is decreasing).

5. Optimization

100 m of fence, a rectangular pen against a wall (three sides fenced). Largest area?

1. **One variable:** with two equal sides x , the third side is $100 - 2x$, so $A(x) = x(100 - 2x) = 100x - 2x^2$, $0 < x < 50$.
2. **Critical points:** $A'(x) = 100 - 4x = 0 \Rightarrow x = 25$.
3. **Justify:** $A''(x) = -4 < 0$ for all x , so A is concave down and $x = 25$ gives the absolute maximum.
4. **Answer the question:** $A(25) = 25 \cdot 50 = 1250 \text{ m}^2$.

Where students lose points

1. Reading the graph of f' as if it were f .
2. Matching graphs by shape instead of by zeros and extrema.
3. Writing “it” or “the graph” without saying which function.
4. An optimization answer with no justification of max or min.
5. Answering with x when the question asks for the area.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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