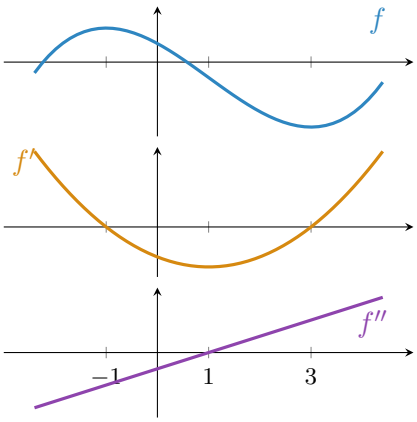
# Connecting $f$, $f'$ and $f''$

AP Calculus AB | Unit 5 | matching graphs, describing $f$ from $f'$, optimization

## 1. Three graphs, lined up




- At $x = -1$ and $x = 3$, $f$ has a max and a min; directly below, $f'$ crosses zero.
- At $x = 1$, $f$ has a point of inflection; $f'$ has its minimum there, and $f''$ crosses zero.
- Where $f$ is increasing, $f'$ is above its axis. Where $f$ is concave up, $f''$ is above its axis.

Each graph tells the slope story of the one above it.

## 2. The dictionary

| $f$ | $f'$ | $f''$ |
|---|---|---|
| increasing | positive | |
| decreasing | negative | |
| relative max / min | changes sign | |
| concave up | increasing | positive |
| concave down | decreasing | negative |
| point of inflection | relative max / min | changes sign |

## 3. Matching unlabeled graphs

**The ten-second method**

Find where one graph has a maximum or minimum. Its derivative must be **zero** at exactly that $x$. Build the chain $f \to f' \to f''$ from there instead of guessing by shape.

## 4. Describing $f$ from the graph of $f'$

Read the graph of $f'$ twice.

- **Above or below the axis** (sign of $f'$): where $f$ is increasing or decreasing, and its relative extrema.
- **Rising or falling** (is $f'$ increasing?): where $f$ is concave up or down, and its points of inflection.

*Example.* At a point where $f'$ is positive and decreasing, $f$ is increasing (because $f' > 0$) and concave down (because $f'$ is decreasing).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## 5. Tables, symmetry, bounds and areas

**A table of $f$.** With $f(3) = 1$, $f(4) = 2$, $f(5) = 9$: $f'(4) \approx \frac{9-1}{5-3} = 4$; the slopes are 1 on $[3, 4]$ and 7 on $[4, 5]$, so $f''(4) \approx \frac{7-1}{1} = 6$.

**Symmetry.** If $f'$ is even and $f(0) = 0$, then $f$ is odd: $f'(x) = 3x^2 - 3$ gives $f(x) = x^3 - 3x$.

**A derivative bound.** $g(1) = 20$ and $g'(x) \leq 5$: by the MVT $g(4) - g(1) = 3g'(c) \leq 15$, so $g(4) \leq 35$.

**Absolute extrema from the graph of $f'$.** Candidates are the endpoints and the critical points; the areas between $f'$ and the axis give the changes in $f$. With $f(0) = 5$, an area of 2 below the axis on $[0, 2]$ and 4 above on $[2, 6]$: $f(2) = 3$, $f(6) = 7$, so the absolute maximum is 7 at $x = 6$.

**A sign change in a table of $f'$.** If a continuous $f'$ goes from 1 to $-3$ on $[3, 4]$, the IVT gives $f'(c) = 0$ in $(3, 4)$, and $f$ has a relative maximum there. If $f'(a) = f'(b)$, the MVT on $f'$ gives $f''(c) = 0$ between them.

## 6. Optimization

*100 m of fence, a rectangular pen against a wall (three sides fenced). Largest area?*

1. **One variable**: with two equal sides $x$, the third side is $100-2x$, so $A(x) = x(100-2x) = 100x-2x^2$, $0 < x < 50$.
2. **Critical points**: $A'(x) = 100 - 4x = 0 \Rightarrow x = 25$.
3. **Justify**: $A''(x) = -4 < 0$ for all $x$, so $A$ is concave down and $x = 25$ gives the absolute maximum.
4. **Answer the question**: $A(25) = 25 \cdot 50 = 1250$ m$^2$.

**Where students lose points**

1. Reading the graph of $f'$ as if it were $f$.
2. Matching graphs by shape instead of by zeros and extrema.
3. Writing "it" or "the graph" without saying which function.
4. An optimization answer with no justification of max or min.
5. Answering with $x$ when the question asks for the area.

**Now practise it**

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