

The Derivative in Context

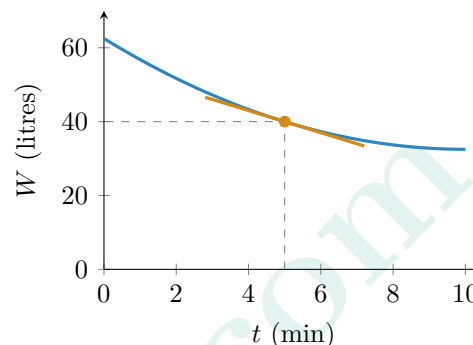
AP Calculus AB | Unit 4 | meaning, units, and the second derivative

1. Value or rate?

$W(t)$ is the number of litres of water in a tank t minutes after a valve opens.

- $W(5) = 40$ is a **value**: the height of the graph.
There are 40 litres in the tank at 5 minutes.
Units: litres.
- $W'(5) = -3$ is a **rate**: the slope of the graph.
It says how fast the amount is changing at that moment. Units: litres per minute.

“The water is -3 at 5 minutes” describes a value, and earns nothing.



2. The three-part sentence

Interpreting $W'(5) = -3$

1. **When:** At $t = 5$ minutes,
2. **What is changing, and which way:** the amount of water in the tank is decreasing (negative $\Rightarrow$ decreasing)
3. **How fast, with units:** at a rate of 3 litres per minute.

Another one: $P(t)$ is the number of people in a store t hours after it opens, and $P'(3) = 20$. At 3 hours after opening, the number of people in the store is increasing at a rate of 20 people per hour.

3. Units

$$\text{units of } f' = \frac{\text{units of } f}{\text{units of } x}$$

$W(t)$ in litres, t in minutes	$W'(t)$ in litres per minute
$C(q)$ in dollars, q in items	$C'(q)$ in dollars per item
$h(t)$ in metres, t in seconds	$h'(t)$ in metres per second

On the free response an interpretation without units does not earn the point.

4. The second derivative in context

$W''(5) = 0.6$ litres per minute per minute: at $t = 5$ minutes, the rate at which the amount of water is changing is increasing by 0.6 litres per minute each minute. The rate goes from -3 toward 0: the tank is still draining, but more slowly.

Two signs, two questions

The sign of f' : is the **quantity** increasing or decreasing?

The sign of f'' : is the **rate** increasing or decreasing?

Here $W' < 0$ and $W'' > 0$: the water is decreasing at an increasing rate.

5. Estimating with a rate

$W(5) = 40$ and $W'(5) = -3$, so one minute later $W(6) \approx 40 + (-3)(1) = 37$ litres (the actual value is 37.3). This is following the tangent line for a short time.

Where students lose points

1. No units, or the units of f instead of the units of f' .
2. No time: “the water decreases by 3 litres per minute”.
3. A value instead of a rate: “the water is -3 ”.
4. “Over the next minute” for an instantaneous rate.
5. Mixing up a decreasing quantity ($f' < 0$) with a decreasing rate ($f'' < 0$).

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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