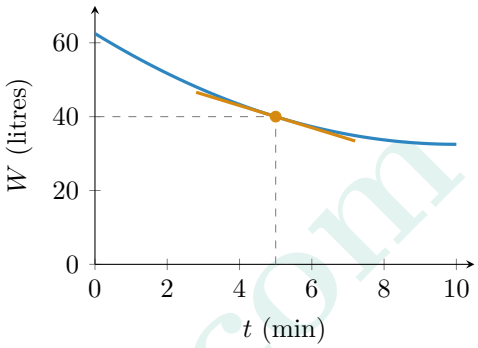
The Derivative in Context

AP Calculus AB | Unit 4 | meaning, units, and the second derivative

1. Value or rate?

$W(t)$ is the number of litres of water in a tank t minutes after a valve opens.

- $W(5) = 40$ is a **value**: the height of the graph. There are 40 litres in the tank at 5 minutes. Units: litres.
- $W'(5) = -3$ is a **rate**: the slope of the graph. It says how fast the amount is changing at that moment. Units: litres per minute.



"The water is -3 at 5 minutes" describes a value, and earns nothing.

2. The three-part sentence

Interpreting $W'(5) = -3$

1. **When**: *At $t = 5$ minutes,*
2. **What is changing, and which way**: *the amount of water in the tank is decreasing* (negative ⇒ decreasing)
3. **How fast, with units**: *at a rate of 3 litres per minute.*

Another one: $P(t)$ is the number of people in a store t hours after it opens, and $P'(3) = 20$. *At 3 hours after opening, the number of people in the store is increasing at a rate of 20 people per hour.*

3. Units

$$\text{units of } f' = \frac{\text{units of } f}{\text{units of } x}$$

$W(t)$ in litres, t in minutes	$W'(t)$ in litres per minute
$C(q)$ in dollars, q in items	$C'(q)$ in dollars per item
$h(t)$ in metres, t in seconds	$h'(t)$ in metres per second

On the free response an interpretation without units does not earn the point.

4. The second derivative in context

$W''(5) = 0.6$ litres per minute *per minute*: *at $t = 5$ minutes, the rate at which the amount of water is changing is increasing by 0.6 litres per minute each minute.* The rate goes from -3 toward 0: the tank is still draining, but more slowly.

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Two signs, two questions

The sign of f': is the **quantity** increasing or decreasing?
The sign of f'': is the **rate** increasing or decreasing?
Here $W' < 0$ and $W'' > 0$: the water is decreasing at an increasing rate.

5. Estimating with a rate

$W(5) = 40$ and $W'(5) = -3$, so one minute later $W(6) \approx 40 + (-3)(1) = 37$ litres (the actual value is 37.3). This is following the tangent line for a short time.

6. Tables, averages, and products

A derivative from a table. $R(t)$, liters of water in a rain barrel: $t = 0, 2, 5, 7, 12$ hours, $R = 10, 16, 23, 26, 33$. Estimate $R'(6)$ with the closest values on either side: $R'(6) \approx \dfrac{R(7) - R(5)}{7 - 5} = \dfrac{26 - 23}{2} = 1.5$ liters per hour. At $t = 6$ hours the water is increasing at about 1.5 liters per hour.

Average rate of change on $[a, b]$ is $\dfrac{R(b) - R(a)}{b - a}$: on $[0, 5]$ it is $\frac{23-10}{5} = 2.6$ L/h, on $[5, 12]$ it is $\frac{33-23}{7} \approx 1.43$ L/h, so the first is greater. An average rate is over an interval; $R'(t)$ is the rate at one instant.

Rate of a product. Revenue $R = p\,q$: with $q(4) = 3$ hundred loaves, $q'(4) = 0.5$, $p(4) = \$6$, $p'(4) = 0.4$: $R'(4) = p'q + pq' = 0.4(3) + 6(0.5) = 4.2$ hundred dollars per week, so at week 4 revenue is increasing at \$420 per week. Not $p'q'$.

Where students lose points

1. No units, or the units of f instead of the units of f'.
2. No time: "the water decreases by 3 litres per minute".
3. A value instead of a rate: "the water is -3".
4. "Over the next minute" for an instantaneous rate.
5. Mixing up a decreasing quantity ($f' < 0$) with a decreasing rate ($f'' < 0$).

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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