

Derivative Rules

AP Calculus AB | Unit 2 | power rule, sin, cos, e^x , $\ln x$

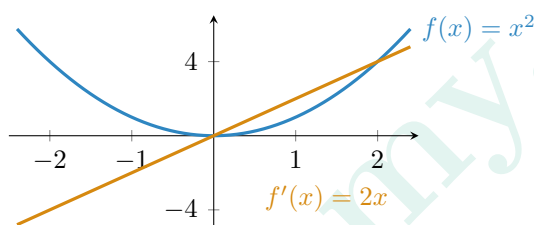
1. The power rule

From the limit definition, $\frac{(x+h)^3 - x^3}{h} = 3x^2 + 3xh + h^2 \rightarrow 3x^2$. The same pattern holds for every power: the exponent comes down in front and drops by one.

Power rule

$$\frac{d}{dx} [x^n] = n x^{n-1} \quad \text{for any real exponent } n.$$

So $\frac{d}{dx} [x^{100}] = 100x^{99}$ — no limit, no expanding.



The slope of x^2 is negative on the left, 0 at the bottom, positive on the right: the slopes trace the line $2x$.

Rewrite as a power first.

$$\frac{d}{dx} [\sqrt{x}] = \frac{d}{dx} [x^{1/2}] = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}, \quad \frac{d}{dx} \left[\frac{1}{x^3} \right] = \frac{d}{dx} [x^{-3}] = -3x^{-4} = -\frac{3}{x^4}.$$

(Careful: $-3 - 1 = -4$, not -2 .)

2. Constants, multiples, sums

$$\frac{d}{dx} [c] = 0, \quad \frac{d}{dx} [c f(x)] = c f'(x), \quad \frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x).$$

A polynomial falls apart term by term: $\frac{d}{dx} [3x^4 - 5x^2 + 7x - 2] = 12x^3 - 10x + 7$.

A number with no x in it is a constant: $\frac{d}{dx} [\pi^2] = 0$, not 2π .

3. Sine, cosine, e^x , $\ln x$

Four derivatives to know by heart

$$\frac{d}{dx} [\sin x] = \cos x, \quad \frac{d}{dx} [\cos x] = -\sin x, \quad \frac{d}{dx} [e^x] = e^x, \quad \frac{d}{dx} [\ln x] = \frac{1}{x} \quad (x > 0).$$

- The slopes of $\sin x$ trace out $\cos x$: slope 1 at 0, slope 0 at the top of the hill, slope -1 at π .
- $\cos x$ starts at its peak and heads downhill, so its derivative carries a minus sign.
- At every point the slope of e^x equals its height. The power rule does *not* apply: in x^n the variable is the base, in e^x it is the exponent.

4. Tangent lines

A tangent line needs a point and a slope. For $f(x) = x^3 - 2x$ at $x = 1$: point $f(1) = -1$; $f'(x) = 3x^2 - 2$, so slope $f'(1) = 1$. Point-slope form (fine to leave on the exam):

$$y - (-1) = 1(x - 1).$$

Where students lose points

1. Not rewriting roots and fractions as powers before using the power rule.
2. Differentiating a constant (π^2 , e^3) as if it contained x : the answer is 0.
3. Losing the minus sign: $(\cos x)' = -\sin x$.
4. Using the power rule on e^x .
5. Negative exponents: $-3 - 1 = -4$, not -2 .

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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