# Derivative Rules

AP Calculus AB | Unit 2 | power rule, sin, cos, $e^x$, $\ln x$

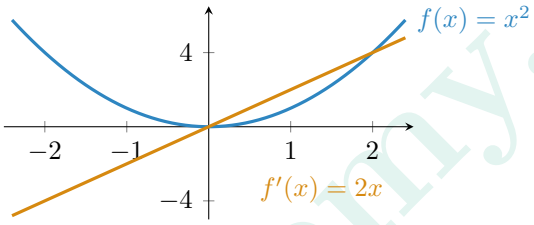
## 1. The power rule

From the limit definition, $\dfrac{(x+h)^3 - x^3}{h} = 3x^2 + 3xh + h^2 \to 3x^2$. The same pattern holds for every power: the exponent comes down in front and drops by one.

**Power rule**

$$\frac{d}{dx}\left[x^n\right] = n\,x^{n-1} \qquad \text{for any real exponent } n.$$

So $\dfrac{d}{dx}\left[x^{100}\right] = 100x^{99}$ — no limit, no expanding.




The slope of $x^2$ is negative on the left, 0 at the bottom, positive on the right: the slopes trace the line $2x$.

**Rewrite as a power first.**

$$\frac{d}{dx}\left[\sqrt{x}\right] = \frac{d}{dx}\left[x^{1/2}\right] = \tfrac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}, \qquad \frac{d}{dx}\left[\frac{1}{x^3}\right] = \frac{d}{dx}\left[x^{-3}\right] = -3x^{-4} = -\frac{3}{x^4}.$$

(Careful: $-3 - 1 = -4$, not $-2$.)

## 2. Constants, multiples, sums

$$\frac{d}{dx}\left[c\right] = 0, \qquad \frac{d}{dx}\left[c\,f(x)\right] = c\,f'(x), \qquad \frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x).$$

A polynomial falls apart term by term: $\dfrac{d}{dx}\left[3x^4 - 5x^2 + 7x - 2\right] = 12x^3 - 10x + 7.$

A number with no $x$ in it is a constant: $\dfrac{d}{dx}\left[\pi^2\right] = 0$, not $2\pi$.

## 3. Sine, cosine, $e^x$, $\ln x$

**Four derivatives to know by heart**

$$\frac{d}{dx}\left[\sin x\right] = \cos x, \qquad \frac{d}{dx}\left[\cos x\right] = -\sin x, \qquad \frac{d}{dx}\left[e^x\right] = e^x, \qquad \frac{d}{dx}\left[\ln x\right] = \frac{1}{x} \quad (x > 0).$$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- The slopes of $\sin x$ trace out $\cos x$: slope 1 at 0, slope 0 at the top of the hill, slope $-1$ at $\pi$.
- $\cos x$ starts at its peak and heads downhill, so its derivative carries a minus sign.
- At every point the slope of $e^x$ equals its height. The power rule does *not* apply: in $x^n$ the variable is the base, in $e^x$ it is the exponent.

## 4. Tangent lines

A tangent line needs a point and a slope. For $f(x) = x^3 - 2x$ at $x = 1$: point $f(1) = -1$; $f'(x) = 3x^2 - 2$, so slope $f'(1) = 1$. Point-slope form (fine to leave on the exam):

$$y - (-1) = 1\,(x - 1).$$

## 5. Horizontal tangents, tan and sec, ln(x + 1)

A horizontal tangent line has slope 0: solve $f'(x) = 0$. For $g(x) = x^2 - 18\ln x$, $g'(x) = 2x - \frac{18}{x} = 0$ gives $x^2 = 9$, so $x = 3$.

$$\frac{d}{dx}\tan x = \sec^2 x, \quad \frac{d}{dx}\sec x = \sec x \tan x, \quad \frac{d}{dx}\cot x = -\csc^2 x, \quad \frac{d}{dx}\csc x = -\csc x \cot x.$$

For $h(x) = 4\tan x - 2\sec x$: $h'(0) = 4(1) - 2(1)(0) = 4$.

$\frac{d}{dx}\ln(x+1) = \frac{1}{x+1}$, so $k(x) = 5\ln(x+1)$ has $k'(4) = 1$. Treat a letter as a constant: $f(x) = x^3 - bx$ gives $f'(2) = 12 - b$, which decreases as $b$ increases.

**Where students lose points**

1. Not rewriting roots and fractions as powers before using the power rule.
2. Differentiating a constant ($\pi^2$, $e^3$) as if it contained $x$: the answer is 0.
3. Losing the minus sign: $(\cos x)' = -\sin x$.
4. Using the power rule on $e^x$.
5. Negative exponents: $-3 - 1 = -4$, not $-2$.

**Now practise it**

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