

Derivative Tests for Extrema

AP Calculus AB | Unit 5 | first derivative test, second derivative test, candidates test

$f'(c) = 0$ only makes c a *candidate*. x^2 , $-x^2$ and x^3 all have $f'(0) = 0$: a minimum, a maximum, and neither. A test decides.

1. The First Derivative Test

At a critical point c

f' changes from $+$ to $-$ at $c \Rightarrow$ relative **maximum**.

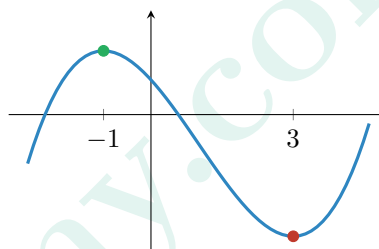
f' changes from $-$ to $+$ at $c \Rightarrow$ relative **minimum**.

No sign change $\Rightarrow$ neither.

$$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 2, \quad f'(x) = (x+1)(x-3).$$

The sign of f' is $+$, $-$, $+$ around -1 and 3 .

" f has a relative maximum at $x = -1$ because f' changes from positive to negative there, and a relative minimum at $x = 3$ because f' changes from negative to positive."



2. The Second Derivative Test

At a point with $f'(c) = 0$

$f''(c) < 0 \Rightarrow$ relative **maximum** (concave down).

$f''(c) > 0 \Rightarrow$ relative **minimum** (concave up).

$f''(c) = 0 \Rightarrow$ the test is **inconclusive**; use the first derivative test.

$$f''(x) = 2x - 2: \quad f''(-1) = -4 < 0 \text{ (maximum)}, \quad f''(3) = 4 > 0 \text{ (minimum)}.$$

With no formula: " g has a relative minimum at $x = 2$ because $g'(2) = 0$ and $g''(2) = 5 > 0$." State **both** facts.

3. Absolute extrema: the Candidates Test

On a closed interval $[a, b]$ the absolute maximum and minimum occur only at critical points or at endpoints. Evaluate f at each candidate and compare. For f on $[-2, 6]$:

x	$f(x)$	
-2	$\frac{4}{3}$	endpoint
-1	$\frac{11}{3}$	critical point
3	-7	critical point $\leftarrow$ absolute minimum
6	20	endpoint $\leftarrow$ absolute maximum

The absolute maximum is at an *endpoint*, not at the relative maximum. The table is the justification.

4. Which test, and what to write

relative extrema, from f' or its sign	First Derivative Test	f' changes from $+$ to $-$
relative extrema, given $f''(c)$	Second Derivative Test	$f'(c) = 0$ and $f''(c) < 0$
absolute extrema on $[a, b]$	Candidates Test	table of endpoints and critical points

“Maximum *value*” asks for $f(c)$; “where” asks for $x = c$.

Where students lose points

1. “ $f'(c) = 0$, so it is a max or a min.”
2. First derivative test without “changes from ... to ...”.
3. Reading $f''(c) = 0$ as “neither” (the test is inconclusive).
4. Forgetting the endpoints for absolute extrema.
5. Giving x when the question asks for the value, or the reverse.

Now practise it

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