# Derivative Tests for Extrema

AP Calculus AB | Unit 5 | first derivative test, second derivative test, candidates test

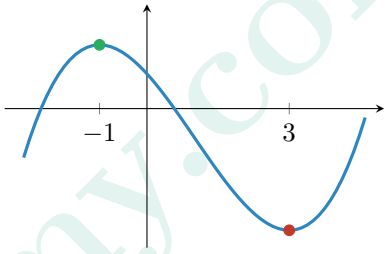
$f'(c) = 0$ only makes $c$ a *candidate*. $x^2$, $-x^2$ and $x^3$ all have $f'(0) = 0$: a minimum, a maximum, and neither. A test decides.

## 1. The First Derivative Test

**At a critical point $c$**

$f'$ changes from $+$ to $-$ at $c \Rightarrow$ relative **maximum**.
$f'$ changes from $-$ to $+$ at $c \Rightarrow$ relative **minimum**.
No sign change $\Rightarrow$ neither.

$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 2$, $f'(x) = (x+1)(x-3)$.
The sign of $f'$ is $+$, $-$, $+$ around $-1$ and $3$.
*"$f$ has a relative maximum at $x = -1$ because $f'$ changes from positive to negative there, and a relative minimum at $x = 3$ because $f'$ changes from negative to positive."*




## 2. The Second Derivative Test

**At a point with $f'(c) = 0$**

$f''(c) < 0 \Rightarrow$ relative **maximum** (concave down).
$f''(c) > 0 \Rightarrow$ relative **minimum** (concave up).
$f''(c) = 0 \Rightarrow$ the test is **inconclusive**; use the first derivative test.

$f''(x) = 2x - 2$: $f''(-1) = -4 < 0$ (maximum), $f''(3) = 4 > 0$ (minimum).

With no formula: *"$g$ has a relative minimum at $x = 2$ because $g'(2) = 0$ and $g''(2) = 5 > 0$."* State **both** facts.

## 3. Absolute extrema: the Candidates Test

On a closed interval $[a, b]$ the absolute maximum and minimum occur only at critical points or at endpoints. Evaluate $f$ at each candidate and compare. For $f$ on $[-2, 6]$:

| $x$ | $f(x)$ | | |
|---|---|---|---|
| $-2$ | $\frac{4}{3}$ | endpoint | |
| $-1$ | $\frac{11}{3}$ | critical point | |
| $3$ | $-7$ | critical point | ← absolute minimum |
| $6$ | $20$ | endpoint | ← absolute maximum |

The absolute maximum is at an *endpoint*, not at the relative maximum. The table is the justification.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## 4. Which test, and what to write

| | | |
|---|---|---|
| relative extrema, from $f'$ or its sign | First Derivative Test | "$f'$ changes from $+$ to $-$" |
| relative extrema, given $f''(c)$ | Second Derivative Test | "$f'(c) = 0$ and $f''(c) < 0$" |
| absolute extrema on $[a, b]$ | Candidates Test | table of endpoints and critical points |

"Maximum *value*" asks for $f(c)$; "where" asks for $x = c$.

## 5. Open intervals, and when only $f'$ is given

**One critical point.** On an open interval there are no endpoints to check. For $g(x) = x + \frac{25}{x}$, $x > 0$: $g'(x) = 1 - \frac{25}{x^2} = 0$ at $x = 5$, and $g'$ changes from negative to positive, so $g$ has a relative minimum there; since $x = 5$ is the *only* critical point on the interval, $g(5) = 10$ is the absolute minimum.

**Only $f'$ and $f(0)$.** Accumulate the change: $f(b) = f(0) + \int_0^b f'(x)\,dx$. With $f'(x) = 6 - 2x$ on $[0, 5]$ and $f(0) = 4$: $f(3) = 4 + 9 = 13$ and $f(5) = 4 + 5 = 9$, so the absolute maximum is 13 (at $x = 3$) and the minimum is 4 (at $x = 0$). A squared factor does not change sign: $f'(x) = (x - 1)^2(x - 4)$ gives no extremum at $x = 1$ and a relative minimum at $x = 4$.

**Where students lose points**

1. "$f'(c) = 0$, so it is a max or a min."
2. First derivative test without "changes from … to …".
3. Reading $f''(c) = 0$ as "neither" (the test is inconclusive).
4. Forgetting the endpoints for absolute extrema.
5. Giving $x$ when the question asks for the value, or the reverse.

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