Exponential Models

AP Calculus AB | Unit 7 | growth and decay from $\frac{dy}{dt} = ky$

1. Proportional to itself

The model

$$\frac{dy}{dt} = ky \quad \Longleftrightarrow \quad y = y_0\, e^{kt}.$$

Separate: $\frac{dy}{y} = k\,dt$, so $\ln|y| = kt + C$ and $y = e^C e^{kt}$, where $e^C = y_0$ is the starting amount. $k > 0$ means growth, $k < 0$ decay, and $k = \dfrac{dy/dt}{y}$ is the relative growth rate.

2. Finding k from data

A culture has 50 cells at $t = 0$ and 400 at $t = 3$ hours. Then $y = 50e^{kt}$ and $400 = 50e^{3k}$, so $e^{3k} = 8$ and

$$k = \frac{\ln 8}{3} = \ln 2, \qquad y = 50\, e^{(\ln 2)t} = 50 \cdot 2^t.$$

Keep k exact until the last line.

- **Doubling time** $= \dfrac{\ln 2}{k}$ (here 1 hour).
- **Half-life** T means $k = -\dfrac{\ln 2}{T}$. With $T = 10$ years, after 25 years $\left(\frac{1}{2}\right)^{25/10} = \left(\frac{1}{2}\right)^{2.5} \approx 0.177$ of the amount is left.

3. How fast, without solving for t

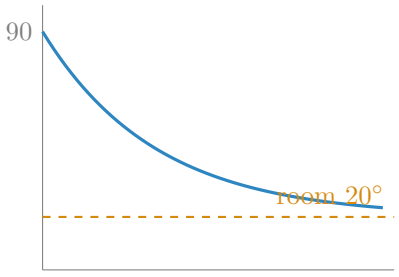
The equation itself gives the rate at any amount: when $y = 300$, $\dfrac{dy}{dt} = ky = (\ln 2)(300) \approx 208$ cells per hour. A question that asks *when* does need t: $50 \cdot 2^t = 1000$ gives $t = \dfrac{\ln 20}{\ln 2} \approx 4.32$ hours.

4. Newton's law of cooling

Soup at 90° in a 20° room: $\dfrac{dT}{dt} = -0.05\,(T - 20)$.
With $u = T - 20$, $\frac{du}{dt} = -0.05\,u$, so the *difference* decays exponentially:

$$T = 20 + 70\, e^{-0.05t}, \qquad T(10) = 20 + 70e^{-0.5} \approx 62.5°.$$

As $t \to \infty$, $T \to 20°$, the room temperature.



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Where students lose points

1. A linear model $y_0 + kt$ instead of $y_0 e^{kt}$.
2. A positive k for a quantity that decays.
3. Reading a half-life T as $k = \frac{1}{T}$ instead of $k = -\frac{\ln 2}{T}$.
4. Rounding k early and carrying the error into every later answer.
5. In cooling problems, treating T itself as exponential; it is $T - T_{\text{room}}$ that decays.

Now practise it

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