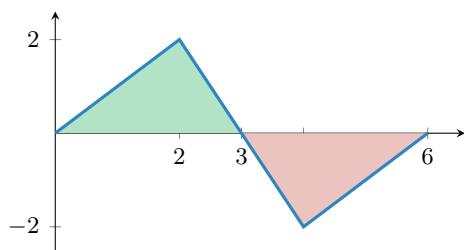


The Fundamental Theorem of Calculus

AP Calculus AB | Unit 6 | signed area, accumulation functions, both parts of the FTC

1. The definite integral is *signed* area



Area above the axis counts as positive, area below as negative. With a graph made of segments, integrate with geometry:

$$\int_0^3 f = 2 + 1 = 3, \quad \int_3^6 f = -3, \quad \int_0^6 f = 0.$$

The last one is the *net* area, not the total area (which is 6).

Properties

$$\begin{aligned} \int_a^a f &= 0, & \int_b^a f &= -\int_a^b f, & \int_a^b f + \int_b^c f &= \int_a^c f, \\ \int_a^b [k f(x) \pm g(x)] dx &= k \int_a^b f(x) dx \pm \int_a^b g(x) dx. \end{aligned}$$

Example. If $\int_0^5 f = 8$ and $\int_3^5 f = 2$, then $\int_0^3 f = 6$ and $\int_3^0 f = -6$.

2. Accumulation functions and FTC Part 1

$g(x) = \int_a^x f(t) dt$ is the area accumulated from a up to x .

Fundamental Theorem of Calculus, Part 1

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

The area grows exactly as fast as the height of f .

So $g' = f$, and all of Unit 5 applies: g is increasing where $f > 0$, has a relative maximum where f changes from $+$ to $-$ (at $x = 3$ above, where $g(3) = 3$), and is concave up where f is increasing.

With a function as the upper limit, use the chain rule: $\frac{d}{dx} \int_0^{x^2} \cos t dt = \cos(x^2) \cdot 2x$.

3. FTC Part 2: evaluating

Fundamental Theorem of Calculus, Part 2

$$\int_a^b f(x) dx = F(b) - F(a) \quad \text{where } F' = f.$$

$$\int_0^2 x^2 dx = \left[\frac{x^3}{3} \right]_0^2 = \frac{8}{3}, \quad \int_1^3 (2x + 1) dx = [x^2 + x]_1^3 = 12 - 2 = 10,$$

$$\int_0^{\pi} \sin x \, dx = [-\cos x]_0^{\pi} = (-\cos \pi) - (-\cos 0) = 1 + 1 = 2.$$

Always upper limit first.

4. Net change

$$\int_a^b f'(x) \, dx = f(b) - f(a) \quad \Longleftrightarrow \quad f(b) = f(a) + \int_a^b f'(x) \, dx.$$

The integral of a rate is the net change: *final value = starting value + accumulated change*. If $f(1) = 3$ and $\int_1^4 f'(x) \, dx = 7$, then $f(4) = 3 + 7 = 10$.

Where students lose points

1. Counting area below the axis as positive.
2. $F(a) - F(b)$: subtracting in the wrong order.
3. Sign slips with $-\cos x$ and negative limits.
4. Forgetting the starting value in $f(b) = f(a) + \int_a^b f'$.
5. $\frac{d}{dx} \int_0^{x^2}$: forgetting the chain-rule factor.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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