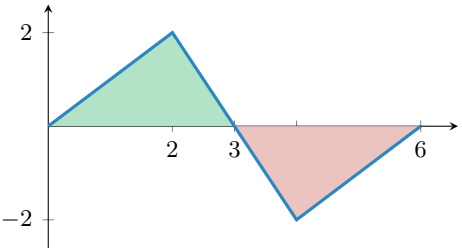
The Fundamental Theorem of Calculus

AP Calculus AB | Unit 6 | signed area, accumulation functions, both parts of the FTC

1. The definite integral is *signed* area



Area above the axis counts as positive, area below as negative. With a graph made of segments, integrate with geometry:

$$\int_0^3 f = 2 + 1 = 3, \quad \int_3^6 f = -3, \quad \int_0^6 f = 0.$$

The last one is the *net* area, not the total area (which is 6).

Properties

$$\int_a^a f = 0, \qquad \int_b^a f = -\int_a^b f, \qquad \int_a^b f + \int_b^c f = \int_a^c f,$$

$$\int_a^b [k\,f(x) \pm g(x)]\,dx = k\int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx.$$

Example. If $\int_0^5 f = 8$ and $\int_3^5 f = 2$, then $\int_0^3 f = 6$ and $\int_3^0 f = -6$.

2. Accumulation functions and FTC Part 1

$g(x) = \int_a^x f(t)\,dt$ is the area accumulated from a up to x.

Fundamental Theorem of Calculus, Part 1

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

The area grows exactly as fast as the height of f.

So $g' = f$, and all of Unit 5 applies: g is increasing where $f > 0$, has a relative maximum where f changes from $+$ to $-$ (at $x = 3$ above, where $g(3) = 3$), and is concave up where f is increasing.

With a function as the upper limit, use the chain rule: $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \cos t\,dt = \cos(x^2)\cdot 2x$.

3. FTC Part 2: evaluating

Fundamental Theorem of Calculus, Part 2

$$\int_a^b f(x)\,dx = F(b) - F(a) \qquad \text{where } F' = f.$$

$$\int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{3}, \qquad \int_1^3 (2x+1)\,dx = \left[x^2 + x\right]_1^3 = 12 - 2 = 10,$$

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$$\int_0^{\pi} \sin x\,dx = \big[-\cos x\big]_0^{\pi} = (-\cos\pi) - (-\cos 0) = 1 + 1 = 2.$$

Always upper limit first.

4. Net change

$$\int_a^b f'(x)\,dx = f(b) - f(a) \qquad \Longleftrightarrow \qquad f(b) = f(a) + \int_a^b f'(x)\,dx.$$

The integral of a rate is the net change: *final value = starting value + accumulated change.* If $f(1) = 3$ and $\int_1^4 f'(x)\,dx = 7$, then $f(4) = 3 + 7 = 10$.

5. Lower limits, substitution, absolute values

Variable in the lower limit. $F(x) = \int_x^6 \sqrt{t^2+7}\,dt = -\int_6^x \sqrt{t^2+7}\,dt$, so $F'(x) = -\sqrt{x^2+7}$ and $F'(3) = -4$.

Substitution. $\int_1^4 f'(2x)\,dx$ with $u = 2x$: $\frac{1}{2}\int_2^8 f'(u)\,du = \frac{1}{2}[f(8) - f(2)]$; with $f(2) = 3$, $f(8) = 11$ this is 4. The limits change with the variable.

Absolute value. Split where the inside changes sign: $\int_{-1}^{5} |x-1|\,dx = \frac{1}{2}(2)(2) + \frac{1}{2}(4)(4) = 10$. If $f \le g$ on $[a,b]$, then $\int_a^b f \le \int_a^b g$.

Where students lose points

1. Counting area below the axis as positive.
2. $F(a) - F(b)$: subtracting in the wrong order.
3. Sign slips with $-\cos x$ and negative limits.
4. Forgetting the starting value in $f(b) = f(a) + \int_a^b f'$.
5. $\dfrac{d}{dx}\int_0^{x^2}$: forgetting the chain-rule factor.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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