The Geometric Distribution

AP Statistics | BITS, the formula, more than / at least / within, mean and SD

1. The geometric setting: BITS

Four conditions

Binary: each trial is a success or a failure.
Independent: one trial does not affect another.
Trials until success: keep going until the *first* success, then stop.
Same probability: p is the same on every trial.

Then X = the number of trials **up to and including** the first success is a geometric random variable. Its values are $1, 2, 3, \ldots$ with no upper limit: it starts at 1, because the success itself counts.

A gardener plants seeds one at a time and stops at the first one that sprouts; each seed sprouts with probability $p = 0.3$, independently. The number of seeds she plants is geometric.

Geometric or not?

- Names drawn from a hat *without replacement* until yours comes up: not geometric, p changes after every draw.
- The number of sprouts among 8 seeds: not geometric, $n = 8$ is fixed, so it is binomial.
- Machine parts tested until the first defective one: geometric. A "success" is simply the event you are waiting for, even when it is something bad.

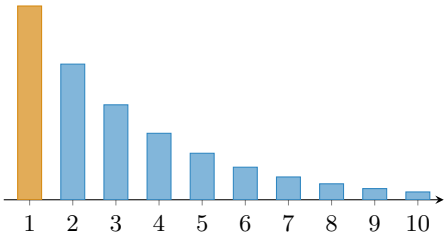
2. The formula

Exactly 3 seeds means fail, fail, sprout. The seeds are independent, so $P(X = 3) = 0.7 \times 0.7 \times 0.3 = 0.147$. There is only **one order**: the success must come last, so there is no binomial coefficient.

$$P(X = k) = (1 - p)^{k-1}\, p$$

($k - 1$ failures, then one success).

Calculator: `geometpdf(p,k)`;
`geometpdf(0.3,3)` $= 0.147$.



seeds planted until the first sprout ($p = 0.3$)

Shape. Each bar is $1 - p = 0.7$ times the bar before it: the probabilities form a geometric sequence, which gives the distribution its name. So the most likely value is always $X = 1$, and the distribution is always **skewed right**, with a long tail of unlucky streaks.

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3. More than, at least, within

Most questions are about a run of failures, not one exact value.

Three shortcuts

$P(X > k) = (1-p)^k$ the first k trials all fail
$P(X \geq k) = (1-p)^{k-1}$ the first $k-1$ trials all fail
$P(X \leq k) = 1-(1-p)^k$ not all of the first k fail ("within k")
More than k and at least k differ by one in the exponent.

In words ($p = 0.3$)	Means	Calculator
exactly 4	$X = 4$	`geometpdf(0.3,4)` $= 0.103$
within the first 4	$X \leq 4$	`geometcdf(0.3,4)` $= 0.760$
fewer than 4	$X \leq 3$	`geometcdf(0.3,3)` $= 0.657$
at least 4	$X \geq 4$	`1 - geometcdf(0.3,3)` $= 0.343 = 0.7^3$
more than 4	$X \geq 5$	`1 - geometcdf(0.3,4)` $= 0.240 = 0.7^4$

`geometcdf(p,k)` always means $P(X \leq k)$.

4. Mean and standard deviation

$$\mu_X = \frac{1}{p}, \qquad \sigma_X = \frac{\sqrt{1-p}}{p}.$$

Here $\mu_X = 1/0.3 \approx 3.33$ seeds and $\sigma_X = \sqrt{0.7}/0.3 \approx 2.79$ seeds: a big spread, often 1 seed and now and then ten or more.

The mean is a long-run average, not a promise. Over many gardeners, the average number of seeds planted until the first sprout is about 3.33. It is larger than the most likely value, 1, because of the long right tail. A higher p means a shorter wait: with $p = 0.6$, $\mu = 1/0.6 \approx 1.67$.

5. Geometric or binomial?

	Binomial	Geometric
X counts	successes in n trials	trials until the first success
values	$0, 1, \ldots, n$	$1, 2, 3, \ldots$
number of trials	fixed n	not fixed
$P(X = k)$	$\binom{n}{k}p^k(1-p)^{n-k}$	$(1-p)^{k-1}p$
mean	np	$1/p$

Same seeds, two questions. How many of 8 seeds sprout? Binomial, $\mu = 8(0.3) = 2.4$ *sprouts*. How many seeds until the first sprout? Geometric, $\mu = 1/0.3 \approx 3.33$ *seeds*. Different questions, different units.

No memory. After 5 failed seeds in a row, the next seed still sprouts with probability 0.3. Independent trials have no memory: a long run of failures does not make a success "due".

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Where students lose points

1. A binomial coefficient in a geometric probability (the success comes last: one order).
2. Off by one: $P(X > k)$ uses k failures, $P(X \geq k)$ uses $k - 1$.
3. Forgetting that X counts the trial with the success too (X starts at 1).
4. Using np for the mean. There is no n: the mean is $1/p$.
5. Calling it geometric when p changes, as when drawing without replacement from a small group.
6. Thinking a success is "due" after a run of failures.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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