Increasing & Decreasing

AP Calculus AB | Unit 5 | the sign of f', sign charts, the graph of f'

1. The sign of f' decides

The rule

$f'(x) > 0$ on an interval $\Rightarrow$ f is **increasing** there.
$f'(x) < 0$ on an interval $\Rightarrow$ f is **decreasing** there.
Only the *sign* of f' matters, never its shape.

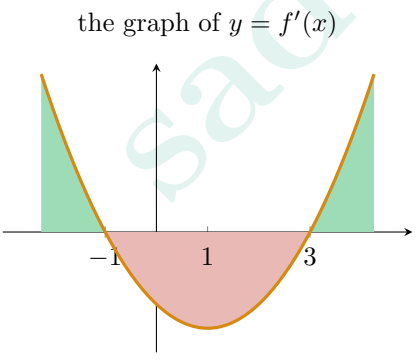
2. The procedure

For $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 2$:

1. Differentiate and factor: $f'(x) = x^2 - 2x - 3 = (x+1)(x-3)$.
2. Find where $f' = 0$ or is undefined: $x = -1$ and $x = 3$. The sign can change only there.
3. Test one point in each interval: $f'(-2) = 5$, $f'(0) = -3$, $f'(4) = 5$.
4. Conclude.

	$x < -1$	$-1 < x < 3$	$x > 3$
$f'(x)$	$+$	$-$	$+$
f	increasing	decreasing	increasing

3. Reading the graph of f'



When the graph shown is f', do **not** ask where it goes up. Ask where it is **above** or **below** the x-axis.

- Above the axis: $f' > 0$, so f is increasing ($x < -1$ and $x > 3$).
- Below the axis: $f' < 0$, so f is decreasing ($-1 < x < 3$).

At $x = 2$ the graph of f' is rising, but $f'(2) = -3 < 0$: f is *decreasing* there. A rising f' is about concavity, not about f increasing.

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4. Writing the justification

What earns the point

"f is increasing on $(-\infty, -1)$ and $(3, \infty)$ because $f'(x) > 0$ on those intervals."

The reason is always a statement about the sign of f', in a sentence. A sign chart alone is scratch work. Open or closed intervals are both accepted at the endpoints.

5. Harder versions

Domain first. $f(x) = \ln(x^2 - 8x + 12)$ exists only where $(x-2)(x-6) > 0$: $x < 2$ or $x > 6$. $f'(x) = \dfrac{2x-8}{x^2-8x+12}$ has the sign of $2x - 8$ on the domain, so f is decreasing on $(-\infty, 2)$ and increasing on $(6, \infty)$.

Shifts and compositions. $g(x) = f(x - c)$ moves every sign change of f' right by c. For $h(x) = f(u(x))$, $h'(x) = f'(u(x)) \cdot u'(x)$: the sign of h' is the product of the signs. If f is increasing everywhere and $g(x) = f(5 - 4x)$, then $g'(x) = -4f'(5 - 4x) < 0$: g is decreasing everywhere.

Parameters. $h(x) = x^3 + bx^2 + 3x$ is increasing for all x when $h'(x) = 3x^2 + 2bx + 3$ never dips below 0: discriminant $(2b)^2 - 36 \le 0$, so $|b| \le 3$. On an interval only, make the smallest value of h' on that interval non-negative: $h(x) = x^3 - 6x^2 + ax$ on $[3, 5]$ has $h'(x) = 3x^2 - 12x + a$ smallest at $x = 3$, so $a - 9 \ge 0$, $a \ge 9$.

Where students lose points

1. Reading where the graph of f' goes up instead of where it is above the axis.
2. No reason, or a reason that never mentions f' ("the graph goes up", "the slope is positive").
3. A sign chart as the whole justification.
4. Forgetting points where f' is undefined.
5. Plugging the test points into f instead of f'.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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