

Independent Events

AP Statistics | Unit 4 | addition rule, independence, multiplication rules

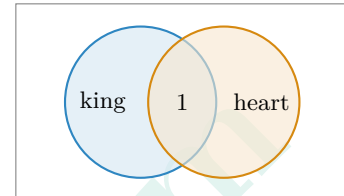
1. A or B: the general addition rule

General addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

“Or” means A, or B, or both.

One card: $P(\text{king or heart}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$. Without the subtraction the king of hearts is counted twice. For mutually exclusive events the overlap is 0 and the rule becomes plain addition.



2. What independent means

Independent events

A and B are **independent** if knowing that one happened does not change the probability of the other:

$$P(A | B) = P(A).$$

How to check: compare two numbers. Among 200 students (100 juniors, 100 seniors):

- $P(\text{sport}) = \frac{70}{200} = 0.35$ but $P(\text{sport} | \text{senior}) = \frac{30}{100} = 0.30$. Different, so *not* independent.
- $P(\text{left-handed}) = \frac{20}{200} = 0.10$ and $P(\text{left-handed} | \text{senior}) = \frac{10}{100} = 0.10$. Equal, so independent.

3. A and B: the multiplication rules

- **General rule** (always true): $P(A \text{ and } B) = P(A) \cdot P(B | A)$.

Two aces without replacement: $\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$.

- **If independent:** $P(A \text{ and } B) = P(A) \cdot P(B)$.

Two sixes on two dice: $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$.

Chains of “and” shrink fast: a machine with 10 independent parts that each work 90% of the time works $0.9^{10} \approx 0.349$ of the time.

4. At least one

$$P(\text{at least one}) = 1 - P(\text{none}).$$

Four rolls of a die: $P(\text{no six}) = \left(\frac{5}{6}\right)^4 \approx 0.482$, so $P(\text{at least one six}) \approx 1 - 0.482 = 0.518$.

5. Mutually exclusive is not independent

Mutually exclusive	Independent
cannot happen together $P(A \text{ and } B) = 0$ if A happened, B is impossible	one tells you nothing about the other $P(A \text{ and } B) = P(A) P(B)$ if A happened, B is exactly as likely

Mutually exclusive events (with nonzero probabilities) are *dependent*: knowing A happened tells you B did not.

Where students lose points

1. Multiplying $P(A) \cdot P(B)$ without independence.
2. Forgetting to subtract $P(A \text{ and } B)$ in an “or” question.
3. Confusing mutually exclusive with independent.
4. Claiming independence without comparing two probabilities.
5. Doing “at least one” case by case instead of $1 - P(\text{none})$.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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