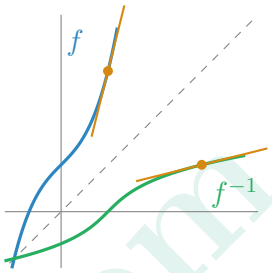
Derivatives of Inverse Functions

AP Calculus AB | Unit 3 | inverse functions and inverse trig functions

1. Reflect the graph, flip the slope

The graph of f^{-1} is the graph of f reflected over $y = x$: the point (a, b) on f becomes (b, a) on f^{-1}. Reflection swaps rise and run, so the slopes at matching points are reciprocals.



The rule

$$\left(f^{-1}\right)'(b) = \frac{1}{f'(a)} \qquad \text{where } f(a) = b.$$

Example. $f(x) = x^3 + x + 1$ has no inverse you could write down, but $f(1) = 3$ and $f'(x) = 3x^2 + 1$, so $f'(1) = 4$ and $\left(f^{-1}\right)'(3) = \frac{1}{4}$.

Why. With $g = f^{-1}$, $f(g(x)) = x$. The chain rule gives $f'(g(x))\,g'(x) = 1$, so $g'(x) = \dfrac{1}{f'(g(x))}$. This needs $f' \neq 0$ at the matching point: a horizontal tangent on f becomes a vertical one on f^{-1}.

2. From a table

x	1	2	4
$f(x)$	2	4	7
$f'(x)$	3	5	6

f is increasing and $g = f^{-1}$. To find $g'(4)$:

1. Where does f *output* 4? $f(2) = 4$, so $g(4) = 2$.
2. $g'(4) = \dfrac{1}{f'(2)} = \dfrac{1}{5}$.

Trap: $\frac{1}{f'(4)} = \frac{1}{6}$ uses the wrong point.

Tangent line to the inverse at $x = 4$: the point on g is $(4, 2)$ and the slope is $\frac{1}{5}$, so $y - 2 = \frac{1}{5}(x - 4)$.

3. arcsin, arccos, arctan

If $y = \arcsin x$, then $\sin y = x$ with $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. Implicit differentiation gives $\cos y \frac{dy}{dx} = 1$, and $\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$ on that interval.

Three to memorise

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}.$$

With the chain rule, multiply by the derivative of the inside and square the *whole* inside:

$$\frac{d}{dx}\arctan\left(x^2\right) = \frac{2x}{1+x^4}, \qquad \frac{d}{dx}\arcsin(3x) = \frac{3}{\sqrt{1-9x^2}}.$$

The slope of $\arctan x$ at $x = \sqrt{3}$ is $\frac{1}{1+3} = \frac{1}{4}$.

sad-academy.com

 © 2026 S&D Academy

Where students lose points

1. Using $f'(b)$ instead of $f'(a)$, where $f(a) = b$.
2. Finding $f'(a)$ and forgetting to take the reciprocal.
3. In a table, looking for b in the x row instead of the $f(x)$ row.
4. Inverse trig: leaving out the chain factor, or writing $1 + 3x^2$ for $1 + (3x)^2$.
5. Mixing up the arcsine and arctangent formulas, or dropping the minus sign of arccosine.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

sad-academy.com

sad-academy.com

AP® is a registered trademark of College Board, which is not affiliated with and does not endorse this material.

 © 2026 S&D Academy