L'Hospital's Rule

AP Calculus AB | Unit 4 | limits of the forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$

1. Zero over zero is not a number

Four limits, each $\frac{0}{0}$ at $x = 0$, four different answers:

$$\lim_{x\to 0}\frac{x}{x} = 1, \qquad \lim_{x\to 0}\frac{3x}{x} = 3, \qquad \lim_{x\to 0}\frac{x^2}{x} = 0, \qquad \lim_{x\to 0}\frac{x}{x^2} \text{ does not exist.}$$

$\frac{0}{0}$ and $\frac{\infty}{\infty}$ are *indeterminate forms*: a race between the top and the bottom, and the limit depends on how fast each one goes.

2. The rule

L'Hospital's rule

If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, or both limits are $\pm\infty$; f and g are differentiable near a with $g'(x) \neq 0$ near a; and the limit on the right exists (or is $\pm\infty$), then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}.$$

The top and the bottom are differentiated *separately*. This is not the quotient rule. The same holds for $x \to \pm\infty$.

$$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{\cos x}{1} = 1.$$

3. Again, and infinity over infinity

If the new limit is still indeterminate, check the form and apply the rule again:

$$\lim_{x\to 0}\frac{e^x - 1 - x}{x^2} = \lim_{x\to 0}\frac{e^x - 1}{2x} = \lim_{x\to 0}\frac{e^x}{2} = \frac{1}{2}, \qquad \lim_{x\to\infty}\frac{x^2}{e^x} = \lim_{x\to\infty}\frac{2x}{e^x} = \lim_{x\to\infty}\frac{2}{e^x} = 0.$$

An exponential outgrows any power of x.

4. When it does not apply

$\lim_{x\to 1}\frac{x^2+5}{x+1} = \frac{6}{2} = 3$ by substitution. It is not an indeterminate form; using the rule anyway gives $\lim_{x\to 1}\frac{2x}{1} = 2$, which is wrong. **Plug in first.**

5. On the free response

Justify the rule before using it: write $\lim_{x\to 0}\sin x = 0$ and $\lim_{x\to 0} x = 0$ separately, then say you are applying L'Hospital's rule. Never write $\frac{0}{0}$ as if it were a value.

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From a table. f and g are differentiable, $f(3) = g(3) = 0$, $f'(3) = 4$, $g'(3) = -2$. Differentiable functions are continuous, so $\lim_{x\to 3} f(x) = 0$ and $\lim_{x\to 3} g(x) = 0$, and

$$\lim_{x\to 3} \frac{f(x)}{g(x)} = \lim_{x\to 3} \frac{f'(x)}{g'(x)} = \frac{4}{-2} = -2.$$

Where students lose points

1. Using the rule when the form is not $\frac{0}{0}$ or $\frac{\infty}{\infty}$.
2. Using the quotient rule instead of $\frac{f'}{g'}$.
3. Not showing that both limits are 0 (or both infinite) on a free response.
4. Writing "$= \frac{0}{0}$" as if it were a number.
5. Stopping while the new limit is still indeterminate.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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