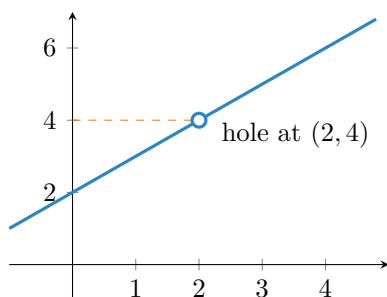


Limits: What They Mean & How to Find Them

AP Calculus AB | Unit 1 | limit notation, one-sided limits, algebraic techniques

1. What a limit means

A limit asks: *as x gets closer and closer to a , what value does $f(x)$ get closer to?* Take $f(x) = \frac{x^2 - 4}{x - 2}$. At $x = 2$ it gives $\frac{0}{0}$, but near $x = 2$ the values close in on 4 from both sides:



x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	3.9	3.99	3.999	?	4.001	4.01	4.1

Notation

$$\lim_{x \rightarrow 2} f(x) = 4 \quad \text{“the limit of } f(x) \text{ as } x \text{ approaches 2 is 4”}$$

The limit is about where f is **heading**, not what $f(2)$ is. If someone defines $f(2) = 1$, the limit is still 4; $f(2)$ can be anything, or not exist at all.

2. One-sided limits

$\lim_{x \rightarrow a^-} f(x)$ is the value approached from the **left**, $\lim_{x \rightarrow a^+} f(x)$ from the **right**. For a jump where the left side heads to 1 and the right side to 3, the two disagree, so $\lim_{x \rightarrow a} f(x)$ **does not exist**.

When does a limit exist?

$$\lim_{x \rightarrow a} f(x) = L \iff \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = L.$$

3. Direct substitution and limit laws

If f has no break at a (polynomials; rational functions with a nonzero denominator; roots, exponentials, sin, cos), just plug in: $\lim_{x \rightarrow 3} (x^2 + 1) = 3^2 + 1 = 10$. This rests on the limit laws:

$$\lim(f \pm g) = \lim f \pm \lim g, \quad \lim(f \cdot g) = \lim f \cdot \lim g, \quad \lim \frac{f}{g} = \frac{\lim f}{\lim g} \quad (\lim g \neq 0).$$

4. When substitution gives $\frac{0}{0}$

$\frac{0}{0}$ is not an answer; it means *simplify first*. Keep writing “lim” on every line until you substitute.

Factor and cancel

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x \rightarrow 2} (x + 2) = 4.$$

Cancelling is allowed because $x \rightarrow 2$ but $x \neq 2$, so $x - 2 \neq 0$. The two functions are the same line except for the one missing point, and the limit ignores that point.

Multiply by the conjugate

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x} = \lim_{x \rightarrow 0} \frac{(x+9) - 9}{x(\sqrt{x+9} + 3)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+9} + 3} = \frac{1}{6}.$$

A limit to know by heart

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad (x \text{ in radians; proved with the squeeze theorem}).$$

Where students lose points

1. Stopping at $\frac{0}{0}$ or writing “undefined”: $\frac{0}{0}$ means do more algebra.
2. Mixing up $f(a)$ and $\lim_{x \rightarrow a} f(x)$.
3. Checking only one side: the limit exists only if both one-sided limits agree.
4. Dropping “lim” before you substitute.
5. Substituting before cancelling, and getting $\frac{0}{0}$ again.

Remember

- A limit is where $f(x)$ is heading as $x \rightarrow a$, not $f(a)$.
- $\lim_{x \rightarrow a} f(x)$ exists $\iff$ the left and right limits are equal.
- Try direct substitution first.
- $\frac{0}{0} \rightarrow$ factor and cancel, or multiply by the conjugate.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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