

Matched Pairs

AP Statistics | Unit 7 | paired data, the paired t test and the paired t interval

Running example. Test scores of 10 students before and after a prep course.

student	1	2	3	4	5	6	7	8	9	10
before	72	85	64	90	78	69	81	75	88	60
after	76	84	70	93	78	74	83	82	86	64
after – before	+4	–1	+6	+3	0	+5	+2	+7	–2	+4

1. What makes data paired

Each value in one group has a natural partner in the other:

- the **same individual** measured twice (before/after, left/right);
- **matched** individuals (twins, subjects paired by age or ability);
- one individual receives **both treatments**, in random order.

The two columns are *not independent*, so a two-sample procedure is wrong. Here it gives $t \approx 0.67$, p-value ≈ 0.25 (“no evidence”), because the large student-to-student variation (scores from 60 to 90) drowns a 3-point improvement. Pairing removes that variation.

2. Work with the differences

Subtract within each pair: 20 numbers become **one sample of 10 differences**, with

$$\bar{d} = 2.8, \quad s_d = 3.01.$$

From here on it is a one-sample t problem about the differences.

3. The paired t test and interval

Paired t procedures for a mean difference

$$t = \frac{\bar{d} - 0}{s_d / \sqrt{n}} \quad \bar{d} \pm t^* \frac{s_d}{\sqrt{n}} \quad df = (\text{number of pairs}) - 1$$

$H_0 : \mu_d = 0$, $H_a : \mu_d > 0$, where μ_d is the true mean difference (after – before) in test score. Always state the order of subtraction.

$$t = \frac{2.8}{3.01 / \sqrt{10}} = \frac{2.8}{0.952} \approx 2.94, \quad df = 9, \quad \text{p-value} = P(t \geq 2.94) \approx 0.008.$$

Because $0.008 < 0.05$, we reject H_0 : there is convincing evidence that scores are higher, on average, after the course.

Interval. $2.8 \pm 2.262(0.952) \Rightarrow (0.65, 4.95)$: we are 95% confident that the true mean improvement in score is between 0.65 and 4.95 points.

4. Conditions, on the differences

- **Random:** a random sample of pairs, or treatments assigned at random within each pair.
- **10%:** the number of pairs is at most 10% of the population (not for experiments).
- **Normal / Large Sample:** at least 30 differences, or a graph of the *differences* with no strong skewness and no outliers.

Paired or two-sample? If each value can be matched with exactly one partner for a reason built into the design, it is paired. Two separate groups (possibly of different sizes) are two-sample.

Where students lose points

1. Running a two-sample t procedure on paired data.
2. Not defining the mean difference, or its order of subtraction.
3. df from the number of measurements instead of the number of pairs.
4. Checking normality on the two columns, not on the differences.
5. Not naming it: paired t test, or paired t interval.

Now practise it

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