The Mean Value Theorem

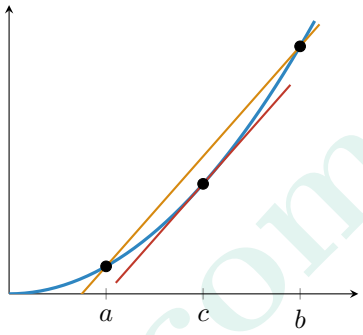
AP Calculus AB | Unit 5 | MVT, Rolle's theorem, Extreme Value Theorem

1. The Mean Value Theorem

MVT

If f is **continuous** on $[a, b]$ and **differentiable** on (a, b), then there is at least one c in (a, b) with

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$



Somewhere, the instantaneous rate of change equals the average rate of change: a tangent line is parallel to the secant line.
$f(x) = x^2$ on $[1, 3]$: $\dfrac{9-1}{3-1} = 4$, and $2c = 4$ gives $c = 2$.

2. Why the conditions matter

$f(x) = |x|$ on $[-1, 1]$: the average slope is 0, but the slope of the graph is only ever -1 or $+1$. There is no such c. The theorem did not fail; it never applied, because f is not differentiable at 0. Check both conditions before using the MVT.

3. Justifying with the MVT

f is differentiable for all x, $f(1) = 4$ and $f(4) = 10$. Must there be a c in $(1, 4)$ with $f'(c) = 2$?

A full-credit answer has three parts

1. **Conditions**: f is differentiable, so it is also continuous on $[1, 4]$.
2. **Average rate of change**: $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{10 - 4}{3} = 2$.
3. **Name and conclusion**: by the Mean Value Theorem there is a c in $(1, 4)$ with $f'(c) = 2$.

The speeding ticket. 10 km in 5 minutes is an average of 120 km/h. Position is continuous and differentiable, so at some instant the velocity was exactly 120 km/h, even though both cameras read 90.

4. Rolle's theorem

Same conditions, and $f(a) = f(b)$. Then $f'(c) = 0$ for some c in (a, b): the MVT with a horizontal secant.

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5. Extreme values and critical points

Extreme Value Theorem

If f is continuous on a **closed** interval $[a, b]$, then f has an absolute maximum and an absolute minimum on it.

A **critical point** is a point c in the domain where $f'(c) = 0$ or $f'(c)$ does not exist. Extreme values occur only at critical points or at endpoints. Not every critical point is an extremum: x^3 has a critical point at 0 and no extremum there.

6. Bounds and impossible claims

Bounds. If $f'(x) \geq 3$ on $(1, 6)$ and $f(1) = 4$, the MVT gives $\frac{f(6)-f(1)}{5} = f'(c) \geq 3$, so $f(6) \geq 19$. If $f'(x) \leq 2$ on $(0, 5)$ and $f(0) = 10$, then $f(5) \leq 10 + 2(5) = 20$: the largest possible value.

Impossible claims. With $g(1) = 2$, $g(5) = 18$ and g differentiable, $g'(c) = \frac{18-2}{4} = 4$ for some c, so "$g'(x) < 3$ everywhere" is impossible.

With a monotone derivative. If f' is strictly increasing on $[2, 6]$, $f(2) = 1$, $f(6) = 9$: $f'(c) = 2$ for some c, and $f'(2) < 2 < f'(6)$.

Where students lose points

1. Using the MVT without stating the conditions.
2. Not naming the theorem.
3. Confusing the MVT (values of f') with the IVT (values of f).
4. Applying it where f has a corner or a jump.
5. "Critical point, so it is an extremum."

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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