Modeling and Verifying Differential Equations

AP Calculus AB | Unit 7 | an equation whose answer is a function

A **differential equation** involves an unknown function and its derivative. Its solution is a *function*, not a number.

1. From words to an equation

Translate piece by piece

the rate of change of A	$\dfrac{dA}{dt}$
is proportional to A	$= kA$
is proportional to the square of v	$= kv^2$
is inversely proportional to x	$= \dfrac{k}{x}$

The constant k is always present; its sign decides growth or decay.

Cooling. A pie cools toward a 20° room at a rate proportional to the difference: $\dfrac{dT}{dt} = -k\,(T - 20)$ with $k > 0$.

Tanks. Rate of change = rate in − rate out: salt enters at 3 kg/min and leaves at $\frac{A}{20}$ kg/min, so $\dfrac{dA}{dt} = 3 - \dfrac{A}{20}$.

2. Verifying a solution

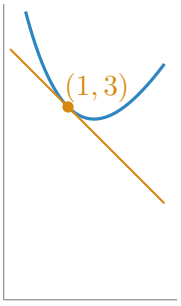
Compute both sides separately and compare them for every x.

- $y = 3e^{2x} - 1$ and $\frac{dy}{dx} = 2y + 2$: the left side is $6e^{2x}$; the right side is $2\,(3e^{2x} - 1) + 2 = 6e^{2x}$. Equal, so it **is** a solution.
- $y = e^{2x} + 1$: the left side is $2e^{2x}$ but the right side is $2e^{2x} + 4$. **Not** a solution.

3. Using the equation without solving it

$\dfrac{dy}{dx} = 2x - y$, and the solution passes through $(1, 3)$.

1. **Slope:** plug in the point: $2(1) - 3 = -1$.
2. **Tangent line estimate:** $f(1.1) \approx 3 + (-1)(0.1) = 2.9$.
3. **Second derivative:** differentiate the equation, then substitute $\frac{dy}{dx}$: $\dfrac{d^2y}{dx^2} = 2 - \dfrac{dy}{dx} = 2 - (2x - y)$, which is $3 > 0$ at $(1, 3)$.
4. Concave up, so the tangent line lies below the curve and 2.9 is an **underestimate**.



sad-academy

 © 2026 S&D Academy

4. Long-run values, families, and e^{kx}

Long run. For $\frac{dQ}{dt} = 12 - 0.4Q$, the level stops changing where $\frac{dQ}{dt} = 0$: $Q = 30$. Below 30 the rate is positive, above 30 negative, so $\lim_{t\to\infty} Q(t) = 30$.

From a family to its equation. $y = x^3 + \frac{C}{x^2}$ gives $y' = 3x^2 - \frac{2C}{x^3}$; eliminating $C = x^2(y - x^3)$ gives $\frac{dy}{dx} = 5x^2 - \frac{2y}{x}$.

Second order. With $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$: $y'' - 2y' - 15y = 0$ becomes $e^{kx}(k^2 - 2k - 15) = 0$, so $k = 5$ or $k = -3$.

Where students lose points

1. Writing $A = kA$ or $y = kA$ instead of $\frac{dA}{dt} = kA$: the equation is about the rate.
2. Dropping k, or giving it the wrong sign for a quantity that decreases.
3. Checking a solution by computing only one side.
4. Differentiating the equation as if the derivative of y were 1: it is $\frac{dy}{dx}$, and it must be substituted.
5. Calling an estimate too high or too low without the sign of $\frac{d^2y}{dx^2}$ at the point.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

sad-academy.com

sad-academy.com

AP® is a registered trademark of College Board, which is not affiliated with and does not endorse this material.

 © 2026 S&D Academy