

Motion with Integrals

AP Calculus AB | Unit 8 | position, velocity, acceleration; displacement vs total distance

1. From velocity back to position

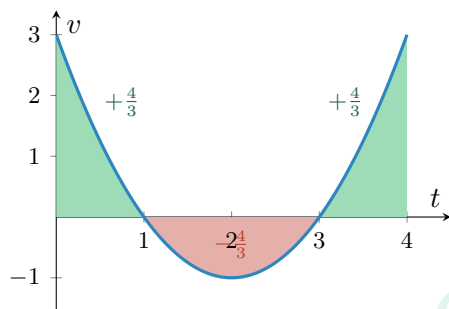
Differentiating goes *down* the ladder $x(t) \rightarrow v(t) \rightarrow a(t)$. Integrating goes back *up*, and you must add the starting value.

New value = starting value + accumulated change

$$x(b) = x(a) + \int_a^b v(t) dt \quad v(b) = v(a) + \int_a^b a(t) dt$$

Example. $a(t) = 6t$, $v(0) = -3$: $v(2) = v(0) + \int_0^2 6t dt = -3 + 12 = 9$.

2. Displacement vs total distance



$v(t) = t^2 - 4t + 3$ on $[0, 4]$; $v = 0$ at $t = 1$ and $t = 3$.

Displacement (signed; areas below the axis are negative):

$$\int_0^4 v(t) dt = \frac{4}{3} - \frac{4}{3} + \frac{4}{3} = \frac{4}{3}.$$

Total distance (integrate the *speed* $|v|$):

$$\int_0^4 |v(t)| dt = \frac{4}{3} + \frac{4}{3} + \frac{4}{3} = 4.$$

If $x(0) = 2$, then $x(4) = 2 + \frac{4}{3} = \frac{10}{3}$: position uses the displacement, never the distance.

By hand: find where $v = 0$, split the integral there, flip the sign of the negative pieces. **Calculator:** integrate **abs(v)** directly.

Another one. $v(t) = \cos t$ on $[0, \pi]$: displacement = 0, total distance = $1 + 1 = 2$.

3. Reading the question

The question says...	you write...
displacement / change in position	$\int_a^b v(t) dt$
total distance travelled	$\int_a^b v(t) dt$
position at time b	$x(a) + \int_a^b v(t) dt$
velocity at time b	$v(a) + \int_a^b a(t) dt$
speed	$ v(t) $

Where students lose points

1. Total distance without the absolute value (that is displacement).
2. Position: forgetting to add the initial position.
3. $|\int v \, dt|$ instead of $\int |v| \, dt$.
4. Using distance, not displacement, to find the position.
5. Differentiating when the question needs an integral.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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