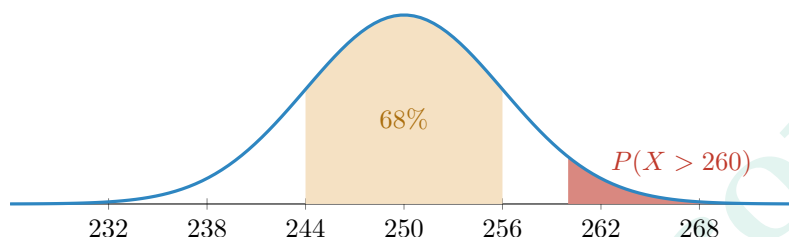


The Normal Distribution

AP Statistics | Unit 1 | 68-95-99.7, z-scores, areas and percentiles

1. The shape and its two numbers

A normal distribution $N(\mu, \sigma)$ is symmetric and single-peaked; mean = median = μ . Changing μ slides the curve; changing σ stretches or squeezes it. The total area under the curve is always 1. Example (hypothetical): a coffee machine fills cups according to $N(250, 6)$ ml; the cups hold 260 ml.



The 68-95-99.7 rule (normal distributions only)

About 68% of values lie within 1σ of the mean (244–256 ml), 95% within 2σ (238–262), 99.7% within 3σ (232–268). So about $(100\% - 95\%)/2 = 2.5\%$ of cups get more than 262 ml.

2. z-scores and areas

$$z = \frac{x - \mu}{\sigma} \quad (\text{how many SDs from the mean}), \quad x = \mu + z\sigma.$$

Overflow: $z = \frac{260 - 250}{6} \approx 1.67$, and $P(X > 260) = P(Z > 1.67) \approx 0.048$ — about one cup in twenty.

Backwards (percentiles). The lightest 10% of cups: the z with area 0.10 to its left is $z \approx -1.28$, so $x = 250 + (-1.28)(6) \approx 242.3$ ml is the 10th percentile.

An answer that earns credit

Not just `normalcdf(260, 1000, 250, 6)`. Show:

$X = \text{amount of coffee in a cup}$, $X \sim N(250, 6)$. $P(X > 260) = P\left(Z > \frac{260-250}{6}\right) = P(Z > 1.67) \approx 0.048$. About 4.8% of cups overflow.

Distribution and parameters, boundary, direction, and the answer in context.

Where students lose points

1. Calculator syntax as the whole answer.
2. Using the 68-95-99.7 rule on data that are not approximately normal.
3. Finding the area on the wrong side (“more than” is the right tail).
4. Flipping the sign of z : it is $(x - \mu)/\sigma$.
5. Using the variance instead of the standard deviation.

Remember

- $N(\mu, \sigma)$: symmetric, single-peaked, total area 1.
- 68% within 1 SD, 95% within 2, 99.7% within 3.
- $z = (x - \mu)/\sigma$; backwards, $x = \mu + z\sigma$.
- Answer = distribution + parameters + boundary + direction + context.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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