

Percentiles, z-Scores & Density Curves

AP Statistics | Unit 1 | measuring position

1. Percentiles

The **p -th percentile** is the value with $p\%$ of the data at or below it. In class B (25 test scores, hypothetical), 12 students scored 80 or less: $12/25 = 48\%$, so 80 is the **48th percentile**. A percentile is *not* a percent correct.

Cumulative relative frequency graph: for each value, the percent of data at or below it. Read across from 50% for the median (about 81), from 25% and 75% for the quartiles (about 75 and 84). Steep stretches mean lots of data packed into a small range.

2. z-scores

$$z = \frac{x - \mu}{\sigma} \quad (\text{how many standard deviations above (+) or below (-) the mean}).$$

Class B's top score, 92: $z = (92 - 76)/14.1 \approx 1.13$, so 1.13 standard deviations above the class mean. z has no units, which makes it the tool for comparing values on different scales.

SAT vs. ACT (illustrative parameters)

Suppose SAT scores have mean 1050, SD 200, and ACT scores mean 21, SD 5.

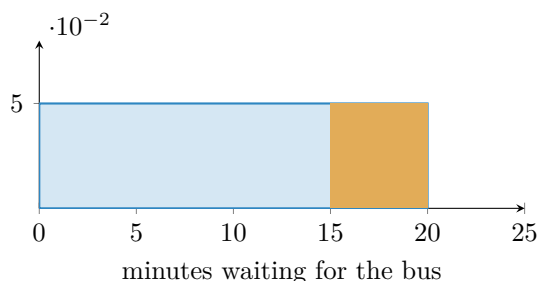
$$z_{SAT} = \frac{1350 - 1050}{200} = 1.5, \quad z_{ACT} = \frac{31 - 21}{5} = 2.$$

The ACT score of 31 is better relative to its test: 2 SDs above the mean versus 1.5.

Changing units (adding a constant, or multiplying by a positive constant) changes x , μ and σ together, so z-scores and percentiles stay the same.

3. Density curves

A **density curve** is always on or above the horizontal axis and has total area exactly 1. The **area** over an interval is the proportion of values in it — area, not height.



A bus comes every 20 minutes: the wait is uniform on $[0, 20]$, height $1/20 = 0.05$, and $P(\text{wait} > 15) = 5 \times \frac{1}{20} = 0.25$.

On any density curve the **median** splits the area in half and the **mean** is the balance point; on a right-skewed curve the mean is pulled to the right of the median.

Where students lose points

1. Treating a percentile as a percent correct.
2. Comparing raw scores on different scales instead of z-scores.
3. A z-score with no interpretation: how many SDs, above or below, of what.
4. Reading a probability from the height of a density curve (it is the area).
5. Using a “density curve” whose total area is not 1.

Remember

- p -th percentile: $p\%$ of the values are at or below it.
- $z = (x - \mu)/\sigma$: SDs above (+) or below (−) the mean.
- Compare across scales with z-scores; changing units does not change z .
- Density curve: area 1, and area = proportion.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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