

Probability Rules

AP Statistics | Unit 4 | simulation, addition rule, conditional probability

1. What probability means

Probability and the law of large numbers

The **probability** of an outcome is the proportion of times it would occur in a very long series of repetitions.

Law of large numbers: the more repetitions, the closer the observed proportion gets to the true probability. It promises nothing about the short run: 7 heads in 10 flips is ordinary for a fair coin, and no outcome is ever “due”.

2. Simulation

When a probability is hard to calculate, estimate it. *A 70% free-throw shooter takes 10 shots. How likely is it that she makes at least 9?*

1. **Model one shot:** a random digit 0–9; digits 0–6 mean a make, 7–9 a miss.
2. **One trial:** read 10 digits and count the makes.
3. **Repeat** many times (here 200 trials).
4. **Estimate:** the proportion of trials in which the event happened.

In 200 trials she made at least 9 shots in 36 of them: an estimate of $36/200 = 0.18$ (the exact value is about 0.149; more trials would get closer). To earn credit, a description must say which digits mean what, what one trial is, and what is counted.

3. The basic rules

- The **sample space** is the list of all possible outcomes (one die: 1, 2, 3, 4, 5, 6).
- Every probability is between 0 and 1, and all outcomes together have probability 1.
- **Complement:** $P(\text{not } A) = 1 - P(A)$. $P(\text{not a six}) = 1 - \frac{1}{6} = \frac{5}{6}$.

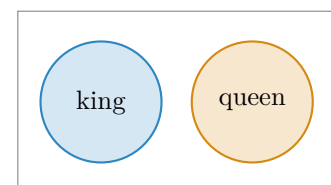
4. Mutually exclusive events

Two events are **mutually exclusive** if they cannot happen together (no outcomes in common). Then

$$P(A \text{ or } B) = P(A) + P(B).$$

One card: $P(\text{king or queen}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$.

“King or heart” is *not* mutually exclusive: the king of hearts is both, and adding would count it twice.



5. Conditional probability

	plays a sport	no sport	total
junior	40	60	100
senior	30	70	100
total	70	130	200

$P(\text{sport}) = \frac{70}{200} = 0.35$. If we are told the student is a senior, only the senior row is left: $P(\text{sport} \mid \text{senior}) = \frac{30}{100} = 0.30$.

Conditional probability

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

“ $\mid$ ” means *given*. The given event is the denominator.

$P(A \mid B) \neq P(B \mid A)$: $P(\text{senior} \mid \text{sport}) = \frac{30}{70} \approx 0.43$, not 0.30.

Where students lose points

1. Confusing $P(A \mid B)$ with $P(B \mid A)$.
2. The wrong denominator in a conditional probability (the grand total instead of the given group).
3. Adding probabilities of events that can overlap.
4. Describing a simulation without a rule for the digits or a definition of one trial.
5. “It’s due”: expecting the short run to even out.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you’ll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

sad-academy.com