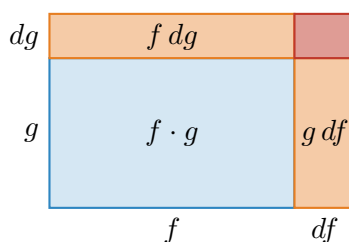


The Product & Quotient Rules

AP Calculus AB | Unit 2 | products, quotients, tan, cot, sec, csc

1. The product rule

The derivative of a product is **not** the product of the derivatives: $\frac{d}{dx}[x^2 \cdot x^3] = \frac{d}{dx}[x^5] = 5x^4$, but $(2x)(3x^2) = 6x^3$.



A rectangle with sides f and g has area fg . Nudge x : the area grows by a strip $g df$, a strip $f dg$, and a corner $df dg$ that is negligible. Dividing by dx :

Product rule

$$(fg)' = f'g + fg'$$

$$\frac{d}{dx} [x^2 \sin x] = 2x \sin x + x^2 \cos x, \quad \frac{d}{dx} [x e^x] = e^x + x e^x.$$

2. The quotient rule

Quotient rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \quad \text{“low d-high minus high d-low, over low squared”}$$

The minus sign makes the order matter: start with the derivative of the *top*. Square the bottom, and leave it squared.

$$\frac{d}{dx} \left[\frac{x^2}{x+1} \right] = \frac{(2x)(x+1) - (x^2)(1)}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2}.$$

3. tan, cot, sec, csc

$$\text{From } \tan x = \frac{\sin x}{\cos x}: \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

$$\begin{aligned} \frac{d}{dx} [\tan x] &= \sec^2 x & \frac{d}{dx} [\cot x] &= -\csc^2 x \\ \frac{d}{dx} [\sec x] &= \sec x \tan x & \frac{d}{dx} [\csc x] &= -\csc x \cot x \end{aligned}$$

Every “co-” function (cos, cot, csc) has a minus sign in its derivative.

4. How the exam asks it

With a table. Given $f(2) = 3$, $f'(2) = -1$, $g(2) = 4$, $g'(2) = 5$:

$$h = fg : h'(2) = (-1)(4) + (3)(5) = 11, \quad k = \frac{f}{g} : k'(2) = \frac{(-1)(4) - (3)(5)}{4^2} = -\frac{19}{16}.$$

Simplify first. $\frac{x^3 + x}{x} = x^2 + 1 \Rightarrow 2x$; and dividing by a constant is not a quotient: $\frac{x^2 + 1}{4} = \frac{1}{4}(x^2 + 1) \Rightarrow \frac{1}{2}x$.

Where students lose points

1. $(fg)' = f'g'$: multiplying the derivatives.
2. The quotient rule in the wrong order (it starts with the derivative of the top).
3. Forgetting to square the bottom.
4. Sign errors with cot, csc, and $-\sin$.
5. Using the quotient rule when simplifying first takes one line.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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