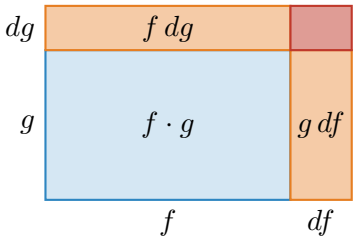
# The Product & Quotient Rules

AP Calculus AB | Unit 2 | products, quotients, tan, cot, sec, csc

## 1. The product rule

The derivative of a product is **not** the product of the derivatives: $\frac{d}{dx}[x^2 \cdot x^3] = \frac{d}{dx}[x^5] = 5x^4$, but $(2x)(3x^2) = 6x^3$.




A rectangle with sides $f$ and $g$ has area $fg$. Nudge $x$: the area grows by a strip $g\,df$, a strip $f\,dg$, and a corner $df\,dg$ that is negligible. Dividing by $dx$:

**Product rule**

$$(fg)' = f'g + fg'$$

$$\frac{d}{dx}\left[x^2 \sin x\right] = 2x \sin x + x^2 \cos x, \qquad \frac{d}{dx}\left[x\,e^x\right] = e^x + x\,e^x.$$

## 2. The quotient rule

**Quotient rule**

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$ "low d-high minus high d-low, over low squared"

The minus sign makes the order matter: start with the derivative of the *top*. Square the bottom, and leave it squared.

$$\frac{d}{dx}\left[\frac{x^2}{x+1}\right] = \frac{(2x)(x+1) - (x^2)(1)}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2}.$$

## 3. tan, cot, sec, csc

From $\tan x = \dfrac{\sin x}{\cos x}$: $\dfrac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \sec^2 x.$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$
$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

Every "co-" function (cos, cot, csc) has a minus sign in its derivative.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## 4. How the exam asks it

**With a table.** Given $f(2)=3$, $f'(2)=-1$, $g(2)=4$, $g'(2)=5$:

$$h=fg:\ h'(2)=(-1)(4)+(3)(5)=11, \qquad k=\frac{f}{g}:\ k'(2)=\frac{(-1)(4)-(3)(5)}{4^2}=-\frac{19}{16}.$$

**Simplify first.** $\dfrac{x^3+x}{x}=x^2+1 \Rightarrow 2x$; and dividing by a constant is not a quotient: $\dfrac{x^2+1}{4}=\frac{1}{4}(x^2+1)\Rightarrow \frac{1}{2}x$.

## 5. Graphs, three factors, working backwards

**From graphs of lines.** The derivative of a line is its slope. $f$ through $(0,1)$ and $(6,4)$: $f(4)=3$, $f'(4)=\frac{1}{2}$; $g$ through $(0,6)$ and $(6,0)$: $g(4)=2$, $g'(4)=-1$. Then $(fg)'(4)=\frac{1}{2}(2)+3(-1)=-2$ and $\left(\frac{f}{g}\right)'(4)=\dfrac{\frac{1}{2}(2)-3(-1)}{2^2}=1$.

**Three factors.** $(uvw)'=u'vw+uv'w+uvw'$. With $f(2)=3$, $f'(2)=-1$, $g(2)=4$, $g'(2)=5$ and $w=x\,f\,g$: $w'(2)=1(3)(4)+2(-1)(4)+2(3)(5)=34$.

**Both rules at once.** $p=\dfrac{x^2 f}{g}$: the top is 12 at $x=2$ and its derivative is $2xf+x^2f'=12-4=8$, so $p'(2)=\dfrac{8(4)-12(5)}{4^2}=-\dfrac{7}{4}$.

**Working backwards.** $h=fg$, $h'(1)=20$, $f(1)=2$, $f'(1)=3$, $g(1)=4$: $20=3(4)+2g'(1)$, so $g'(1)=4$. Also $(f^2)'=2ff'$; and in $f'g+fg'$ the $g'$ term is multiplied by $f$, so if $f>0$ a larger $g'$ gives a larger $h'$.

### Where students lose points

1. $(fg)'=f'g'$: multiplying the derivatives.
2. The quotient rule in the wrong order (it starts with the derivative of the top).
3. Forgetting to square the bottom.
4. Sign errors with cot, csc, and $-\sin$.
5. Using the quotient rule when simplifying first takes one line.

### Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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