

Random Variables: Mean & SD

AP Statistics | Unit 4 | probability distributions, expected value, standard deviation

1. Random variable and its distribution

A **random variable** is a number that depends on the outcome of a chance process. Its **probability distribution** lists every possible value with its probability.

A carnival game costs \$2. Let X be the amount won on one spin.

winnings x	\$0	\$2	\$5	\$20
probability	0.60	0.25	0.12	0.03

Every probability is between 0 and 1, and they add to 1 ($0.60 + 0.25 + 0.12 + 0.03 = 1$). Check this first; a missing probability is found as 1 minus the rest.

Discrete: a list of separate values, described by a table. **Continuous:** any value in an interval, described by a density curve.

2. The mean (expected value)

Mean of a discrete random variable

$$\mu_X = E(X) = \sum x_i p_i$$

$$\mu_X = (0)(0.60) + (2)(0.25) + (5)(0.12) + (20)(0.03) = 0 + 0.50 + 0.60 + 0.60 = \$1.70.$$

You pay \$2 and get back \$1.70 on average: a loss of 30 cents a game.

Interpretation. If you play this game many, many times, the average amount won per game will be about \$1.70. Three ingredients: the long run, the word “average”, and the context. The mean need not be a possible value: nobody wins \$1.70 on a spin.

3. The standard deviation

Standard deviation of a discrete random variable

$$\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 p_i} \quad (\text{the sum under the root is the variance})$$

x	p	$x - \mu$	$(x - \mu)^2 p$
0	0.60	-1.70	1.734
2	0.25	0.30	0.023
5	0.12	3.30	1.307
20	0.03	18.30	10.047
variance			13.11

$\sigma_X = \sqrt{13.11} \approx \$3.62$. The rare \$20 prize contributes most of the spread.

Interpretation. *The amount won on a spin typically varies by about \$3.62 from the mean of \$1.70.*

4. Reading probabilities from the table

$$P(X \geq 5) = 0.12 + 0.03 = 0.15. \quad P(X > 0) = 1 - P(X = 0) = 1 - 0.60 = 0.40.$$

“At least 5” includes 5; “more than 5” does not.

Where students lose points

1. Interpreting the mean without “long run” or “average”.
2. Expecting the mean to be one of the possible values.
3. Reporting the variance when the standard deviation is asked.
4. Not checking that the probabilities add to 1.
5. Reading “at least” as “more than”.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

sad-academy.com

The game and its probabilities are hypothetical. AP® is a registered trademark of College Board, which is not affiliated with and does not endorse this material.