

Riemann Sums

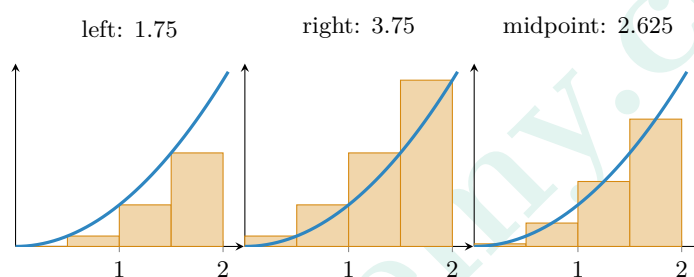
AP Calculus AB | Unit 6 | accumulation, left/right/midpoint/trapezoid, tables

1. Area is accumulated change

At a constant 30 m/s for 4 s, distance = $30 \times 4 = 120$ m: height $\times$ width, the *area* under the velocity graph. In general, **the area under a rate graph is the accumulated change**, with units (rate units) $\times$ (input units): $\frac{\text{m}}{\text{s}} \times \text{s} = \text{m}$.

2. Left, right, midpoint, trapezoid

Cut $[a, b]$ into subintervals and add **width** $\times$ **height** for each. For $f(x) = x^2$ on $[0, 2]$ with $n = 4$ ($\Delta x = 0.5$):



- **Left:** $0.5 [f(0) + f(0.5) + f(1) + f(1.5)] = 1.75$
- **Right:** $0.5 [f(0.5) + f(1) + f(1.5) + f(2)] = 3.75$
- **Midpoint:** $0.5 [f(0.25) + f(0.75) + f(1.25) + f(1.75)] = 2.625$
- **Trapezoidal:** the average of left and right, $\frac{1}{2}(1.75 + 3.75) = 2.75$

The exact area is $\frac{8}{3} \approx 2.667$.

3. Over or under?

No exact answer needed

f increasing: left sum is an underestimate, right sum is an overestimate.

f decreasing: left sum is an overestimate, right sum is an underestimate.

Trapezoidal sum: **concave up** $\Rightarrow$ overestimate; **concave down** $\Rightarrow$ underestimate.

Give the reason: “the right Riemann sum is an overestimate because f is increasing.”

4. From a table, with unequal widths

t (s)	0	2	5	9
$v(t)$ (m/s)	20	32	44	52

Four values make **three** subintervals, of widths 2, 3 and 4. Multiply each width by its own height:

- Left: $(2)(20) + (3)(32) + (4)(44) = 312$ m
- Right: $(2)(32) + (3)(44) + (4)(52) = 404$ m
- Trapezoidal: $\frac{1}{2}(312 + 404) = 358$ m

5. More rectangles: the definite integral

For x^2 on $[0, 2]$ the right sums are 3.750, 3.188, 2.922, 2.793, 2.729 for $n = 4, 8, 16, 32, 64$: they close in on $\frac{8}{3}$.

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

Where students lose points

1. Using one Δx when the table has unequal widths.
2. Mixing up left and right, or using one rectangle too many.
3. Over/under with no reason.
4. Judging a trapezoidal sum by increasing/decreasing instead of concavity.
5. No units on an accumulation answer.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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