

Sampling Distribution of a Mean

AP Statistics | Unit 5 | centre, spread and shape of $\bar{x}$

Centre, spread, shape

Centre: $\mu_{\bar{x}} = \mu$ ($\bar{x}$ is an unbiased estimator of μ).

Spread: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, provided $n \leq 10\%$ of the population.

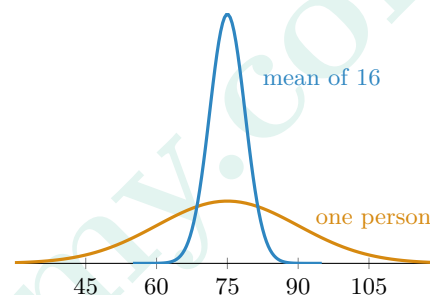
Shape: normal if the population is normal (any n); approximately normal if $n \geq 30$ (Central Limit Theorem); otherwise a normal model cannot be assumed.

1. Averages vary less than individuals

Adult weights: $\mu = 75$ kg, $\sigma = 15$ kg. For the mean weight of $n = 16$ random adults,

$$\sigma_{\bar{x}} = \frac{15}{\sqrt{16}} = 3.75 \text{ kg.}$$

Same centre, four times less spread: in a group a heavy person is usually balanced by a light one. With $n = 64$ the spread is 1.875 kg: four times the sample halves it.



2. When is $\bar{x}$ normal?

the population is normal	yes, for <i>any</i> sample size
population not normal, $n \geq 30$	yes, approximately (Central Limit Theorem)
population not normal, $n < 30$	no: a normal model cannot be assumed

Always give the reason: “the population is normal” or “ $n \geq 30$, so the CLT applies”.

3. One person vs the average of 16

An elevator holds 16 people and 1300 kg: $1300/16 = 81.25$ kg per person. Weights are approximately normal with $\mu = 75$, $\sigma = 15$.

one person over 81.25 kg	$z = \frac{81.25 - 75}{15} \approx 0.42$	$P \approx 0.338$
the mean of 16 over 81.25 kg	$z = \frac{81.25 - 75}{3.75} \approx 1.67$	$P \approx 0.048$

Read the question first

About **one individual**: divide by σ . About a **sample mean**: divide by $\frac{\sigma}{\sqrt{n}}$.

4. A difference of two means

For independent random samples from two populations:

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2, \quad \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

The variances add, even for a difference. Example: $\mu_1 = 75$, $\sigma_1 = 15$, $n_1 = 25$ and $\mu_2 = 70$, $\sigma_2 = 12$, $n_2 = 36$ give mean 5 and standard deviation $\sqrt{9 + 4} \approx 3.61$.

Where students lose points

1. Dividing by σ when the question is about a mean.
2. Dividing by $\sigma/\sqrt{n}$ when it is about one individual.
3. A normal model with a small sample from a skewed population.
4. “Approximately normal” with no reason given.
5. Subtracting variances for a difference of means.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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