

Significance Test for a Mean

AP Statistics | Unit 7 | the one-sample t test: hypotheses, t , p-value, conclusion

Running example. A phone maker claims a mean battery life of 9 hours. A random sample of 20 phones has $\bar{x} = 8.4$ h and $s = 1.2$ h. Is there convincing evidence that the true mean is less than 9 hours?

1. Hypotheses about a mean

$$H_0 : \mu = 9 \qquad H_a : \mu < 9$$

where μ is the true mean battery life of *all* phones of this model.

- Hypotheses are about the parameter μ , never about $\bar{x}$.
- The value in H_0 comes from the claim (9), not from the sample (8.4).
- $<$, $>$ or $\neq$ is set by the question, before looking at the data.

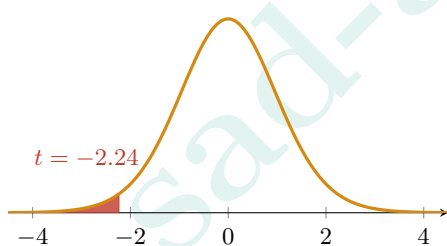
2. The t statistic

One-sample t test for a mean

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \quad \text{with } df = n - 1$$

$\frac{s}{\sqrt{n}} = \frac{1.2}{\sqrt{20}} \approx 0.268$, so $t = \frac{8.4 - 9}{0.268} \approx -2.24$ with $df = 19$: the sample mean is 2.24 standard errors below the claim.

3. The p-value and the conclusion



The p-value is the tail area in the direction of H_a under the t curve with $df = 19$:

$$P(t \leq -2.24) \approx 0.019 \quad (\text{tcdf}).$$

If the true mean really were 9 hours, a sample mean of 8.4 or less would occur only about 1.9% of the time.

Conclusion. Because the p-value $0.019 < \alpha = 0.05$, we reject H_0 . There is convincing evidence that the true mean battery life of all phones of this model is less than 9 hours.

For a two-sided alternative ($H_a : \mu \neq 9$) both tails count: p-value $\approx 2(0.019) \approx 0.038$.

4. The full answer

1. **Hypotheses**, with μ defined in context.
2. **Name** the procedure (one-sample t test for a mean) and check the **conditions**: Random; 10%; Normal / Large Sample ($n \geq 30$, or for $n < 30$ a graph of the data with no strong skewness and no outliers).

3. **Mechanics:** t , df , p-value.
4. **Conclusion:** compare the p-value to α , decide about H_0 , state it in context.

A large p-value means *fail to reject* H_0 (not enough evidence for H_a); never “accept H_0 ”. A two-sided test at $\alpha = 0.05$ rejects H_0 exactly when the 95% confidence interval does not contain μ_0 .

Where students lose points

1. Hypotheses about $\bar{x}$, or μ never defined.
2. Using z and `normalcdf` instead of t and `tcdf`.
3. Skipping the conditions, or no graph for a small sample.
4. “Accept H_0 ”, or “this proves”.
5. A conclusion with no link to α , or with no context.

Now practise it

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