

Significance Test for a Proportion

AP Statistics | Unit 6 | hypotheses, test statistic, p-value, conclusion

1. The logic

1. **Assume** nothing is going on (the null hypothesis).
2. Ask: **how surprising** would our data be if that were true?
3. Very surprising $\Rightarrow$ doubt the assumption. Not surprising $\Rightarrow$ no reason to doubt it.

Running example. A coin is spun on its edge 200 times and lands heads 84 times ($\hat{p} = 0.42$). Is a spun coin unfair?

2. The hypotheses

Null and alternative

$$H_0 : p = 0.5$$

$$H_a : p \neq 0.5$$

where p = the true proportion of heads when this coin is spun.

- Hypotheses are about the **parameter** p , never about $\hat{p}$. Define p in words.
- H_0 always has “=”.
- H_a is $<$, $>$ or $\neq$, decided by the question *before* seeing the data. (“The company claims 60%; you suspect it is lower”: $H_0 : p = 0.60$, $H_a : p < 0.60$.)

3. Conditions and the test statistic

Assume H_0 is true, so use the null value p_0 (not $\hat{p}$):

- **Random:** 200 spins of the coin.
- **Large Counts:** $np_0 = 200(0.5) = 100 \geq 10$ and $n(1 - p_0) = 100 \geq 10$.
- (10% condition when sampling without replacement from a population.)

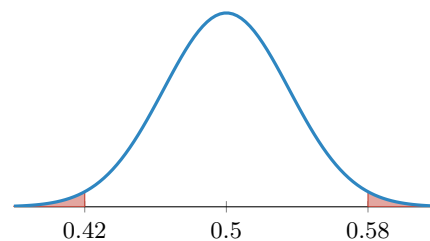
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = \frac{0.42 - 0.50}{0.0354} \approx -2.26.$$

Name the procedure: *one-sample z test for a proportion.*

4. The p-value

Definition

The **p-value** is the probability of getting a result at least as extreme as the one observed, *assuming the null hypothesis is true*.



Two-sided, so both tails count: $2 \times P(z \leq -2.26) = 2(0.0118) \approx 0.024$.

Interpretation. If the coin were fair, there would be about a 2.4% chance of getting a sample proportion as far from 0.5 as 0.42, or farther, in 200 spins. It is **not** the probability that H_0 is true.

5. The conclusion

p-value $< \alpha$ **reject** H_0 : convincing evidence for H_a

p-value $\geq \alpha$ **fail to reject** H_0 : not convincing evidence for H_a

“Because the p-value of 0.024 is less than $\alpha = 0.05$, we reject H_0 . There is convincing evidence that the true proportion of heads when this coin is spun is not 0.5.” Three parts: the comparison, the decision, the conclusion in context about H_a .

Never write “accept H_0 ” or “ H_0 is true”: not enough evidence against something is not evidence for it.

Where students lose points

1. Hypotheses about $\hat{p}$ instead of p , or p never defined.
2. “The p-value is the probability that H_0 is true.”
3. “Accept H_0 ” or “this proves H_0 ”.
4. Using $\hat{p}$ instead of p_0 in the conditions and the standard deviation.
5. A conclusion with no comparison to α , or no context.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you’ll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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