# Slope Fields

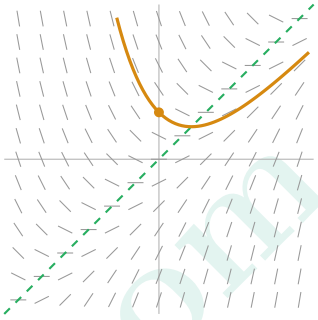
AP Calculus AB | Unit 7 | every solution, before solving

## 1. Drawing a slope field

At each point $(x, y)$ of a grid, the differential equation gives a slope. Draw a short segment with that slope there. For $\dfrac{dy}{dx} = x - y$:

$$(0,0) : 0, \quad (1,0) : 1, \quad (0,1) : -1, \quad (-1,1) : -2.$$

Shortcut: the slope is 0 wherever the right side is 0, so every segment on the line $y = x$ is flat.
Each segment is a piece of the tangent line to the solution through that point.



## 2. Matching a field to its equation

**Three tests**

1. **Columns.** Segments parallel down each vertical column: the slope depends only on $x$, as in $\frac{dy}{dx} = x$.
2. **Rows.** Segments parallel along each horizontal row: the slope depends only on $y$, as in $\frac{dy}{dx} = y$.
3. **Zeros and signs.** Find where the segments are flat (the right side is 0) and check the sign of the slope in one region.

## 3. Sketching a solution through a point

Start at the given point and move left and right, staying tangent to the segments, never cutting across them. Solutions of these equations do not cross. For $\frac{dy}{dx} = x - y$ the solutions are $y = x - 1 + Ce^{-x}$; the one through $(0, 1)$ dips, then climbs toward the straight-line solution $y = x - 1$.

## 4. Equilibrium and the long run

When the slope depends only on $y$, a row where the slope is 0 for every $x$ gives a constant solution, an **equilibrium**.

- $\frac{dy}{dx} = 0.5\,(4 - y)$: equilibrium $y = 4$. Slopes are negative above and positive below, so solutions approach 4: **stable**.
- $\frac{dy}{dx} = y - 3$: equilibrium $y = 3$. Solutions move away from it: **unstable**.


© 2026 S&D Academy

## 5. Classifying a solution, and shifting it

Without solving: for $\frac{dy}{dx} = x - y$, differentiate to get $\frac{d^2y}{dx^2} = 1 - \frac{dy}{dx}$. Through $(2, 2)$: $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 1 > 0$, a relative minimum. Through $(0, 2)$: $\frac{dy}{dx} = -2$ and $\frac{d^2y}{dx^2} = 3 > 0$, decreasing and concave up.

For an autonomous equation (the slope depends only on $y$, e.g. $\frac{dy}{dx} = 0.5(4 - y)$) every row of the slope field is the same, so a horizontal shift of a solution is again a solution: if $y = f(x)$ solves it, so does $y = f(x - 2)$.

**Where students lose points**

1. Computing a slope with $x$ and $y$ swapped.
2. Drawing a solution curve that cuts across the segments instead of following them.
3. Mixing up columns and rows when matching a field to its equation.
4. Missing a constant solution, where the slope is 0 for every $x$.
5. Reversing stable and unstable: check the sign of the slope on both sides.

**Now practise it**

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

**sad-academy.com**

sad-academy.com

AP® is a registered trademark of College Board, which is not affiliated with and does not endorse this material.

 © 2026 S&D Academy