Straight-Line Motion

AP Calculus AB | Unit 4 | position, velocity, acceleration

1. Position, velocity, acceleration

$x(t)$	**position**: where the particle is	metres
$v(t) = x'(t)$	**velocity**: how fast, and which way	metres per second
$a(t) = v'(t) = x''(t)$	**acceleration**: how the velocity changes	metres per second^2

Example used throughout, for $0 \le t \le 4$:

$$x(t) = t^3 - 6t^2 + 9t, \qquad v(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3), \qquad a(t) = 6t - 12.$$

2. Direction: the sign of velocity

- $v > 0$: moving in the positive direction (right). $v < 0$: moving in the negative direction (left).
- The particle **changes direction** where $v = 0$ *and* v changes sign.

Here $v = 0$ at $t = 1$ and $t = 3$, and v changes sign at both: the particle turns around twice.

3. Speeding up or slowing down?

speed $= |v(t)|$. Velocity has a sign (a direction); speed does not.

Compare the signs of v and a

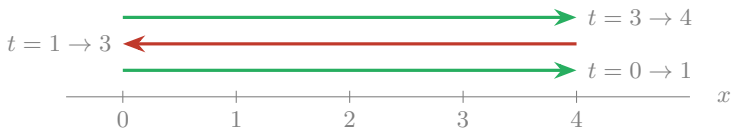
v and a have the **same sign** ⇒ speeding up.
v and a have **opposite signs** ⇒ slowing down.

The sign of a alone tells you nothing: $v = -5$, $a = -2$ is *speeding up* (moving left and being pushed left).

	$0 < t < 1$	$1 < t < 2$	$2 < t < 3$	$3 < t < 4$
$v(t)$	+	−	−	+
$a(t)$	−	−	+	+
speed	slowing down	**speeding up**	slowing down	**speeding up**

On the free response give the reason: *"the particle is speeding up because v and a have the same sign."*

4. Displacement vs total distance



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- **Displacement**: net change in position. $x(4) - x(0) = 4 - 0 = 4$.
- **Total distance**: everything travelled. Split at every turn ($v = 0$) and add the pieces as positives: $|x(1) - x(0)| + |x(3) - x(1)| + |x(4) - x(3)| = 4 + 4 + 4 = 12$.

5. Greatest speed, farthest point, average velocity

Greatest speed. Check the candidates: where $a = 0$ (an extreme of v, not a stop) and the endpoints, then compare $|v|$. For $v(t) = t^2 - 8t + 7$ on $[0, 6]$: $a = 2t - 8 = 0$ at $t = 4$; $v(0) = 7$, $v(4) = -9$, $v(6) = -5$, so the greatest speed is 9 at $t = 4$.

Farthest point. For $x(t) = 2t^3 - 15t^2 + 24t$ on $[0, 5]$: $v = 6(t - 1)(t - 4)$; $x(0) = 0$, $x(1) = 11$, $x(4) = -16$, $x(5) = -5$, so farthest left is -16 at $t = 4$ and farthest right 11 at $t = 1$. Average velocity on $[1, 3]$ is $\frac{x(3)-x(1)}{2} = \frac{-9-11}{2} = -10$.

Rate of change of speed. Speed $= |v|$ changes at rate a when $v > 0$ and $-a$ when $v < 0$. At $t = 2$: $v = -12$, $a = -6$, so speed rises at 6 units/s^2 (same signs: speeding up).

Where students lose points

1. "$a < 0$, so it is slowing down." Compare the signs of v and a.
2. Treating speed and velocity as the same thing.
3. Calling $v = 0$ a turn without checking that v changes sign.
4. Giving the displacement when total distance is asked.
5. A conclusion with no reason.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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