Tangent Line Approximation

AP Calculus AB | Unit 4 | local linearity and linearization

1. Local linearity

Zoom in on a differentiable curve near a point and it becomes indistinguishable from its tangent line there. So near $x = a$ we may use the line, which is easy to compute, in place of the curve.

2. The tangent line approximation

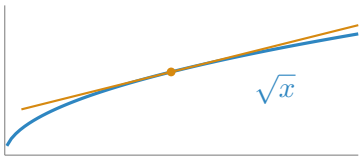
Linearization at $x = a$

$$f(x) \approx L(x) = f(a) + f'(a)(x - a) \qquad \text{for } x \text{ near } a.$$

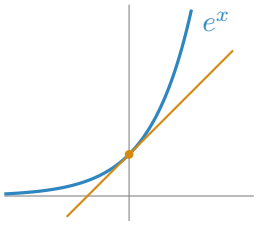
A point you know, plus the slope there times how far you move. Choose a close to the x you want, with $f(a)$ and $f'(a)$ easy to find.

Example. $\sqrt{4.1}$: take $f(x) = \sqrt{x}$, $a = 4$, so $f(4) = 2$ and $f'(x) = \dfrac{1}{2\sqrt{x}}$, $f'(4) = \frac{1}{4}$. Then $\sqrt{4.1} \approx 2 + \frac{1}{4}(0.1) = 2.025$ (calculator: 2.02485).

3. Too high or too low?



Concave down ($f'' < 0$): the tangent line lies *above* the curve, so $L(x)$ **overestimates**.



Concave up ($f'' > 0$): the tangent line lies *below* the curve, so $L(x)$ **underestimates**. Example: $e^{0.1} \approx 1 + 0.1 = 1.1 < 1.10517$.

On a free response the reason is the **sign of** f'' near a, not "the graph looks curved".

4. No formula for f? One value and one slope are enough

From f'. $f(1) = 3$ and $f'(x) = x^3 + 1$, so $f'(1) = 2$ and $f(1.1) \approx 3 + 2(0.1) = 3.2$. The slope comes from the point you *know* ($x = 1$), not from the point you want.

From a table. $W(t)$ litres of water in a tank, t minutes in: $W(2) = 40$, $W'(2) = -6$.

$$W(2.3) \approx W(2) + W'(2)(2.3 - 2) = 40 + (-6)(0.3) = 38.2 \text{ litres.}$$

5. Implicit curves, compositions, and zeros

Implicit curve. On $x^2 + xy + y^2 = 19$ at $(2, 3)$: $y' = -\frac{2x+y}{x+2y} = -\frac{7}{8}$, so $y(2.2) \approx 3 - \frac{7}{8}(0.2) = 2.825$.

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Composition. With $f(9) = 4$, $f'(9) = 2$ and $g(x) = f(x^2)$: $g(3) = 4$, $g'(3) = 2(3)f'(9) = 12$, so $g(3.1) \approx 4 + 12(0.1) = 5.2$.

A zero. Set the tangent line to 0: $f(3) = 1.2$, $f'(3) = -0.6$ give $1.2 - 0.6(x - 3) = 0$, $x \approx 5$. **A rate.** Use the tangent line of f' (slope f''): $f'(2) = 5$, $f''(2) = -1.5$ give $f'(2.4) \approx 4.4$.

Where students lose points

1. Using the slope at the wrong point: $f'(x)$ instead of $f'(a)$.
2. Dropping $f(a)$ and writing only $f'(a)(x - a)$.
3. The wrong sign of $x - a$, or ignoring a negative slope.
4. Calling an estimate too high or too low without naming the sign of f''.
5. Writing $=$ instead of $\approx$ for the estimate.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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