

Type I & Type II Errors

AP Statistics | Unit 6 | the two errors, their probabilities, and power

1. Two ways to be wrong

	H_0 is true	H_0 is false
reject H_0	Type I error	correct
fail to reject H_0	correct	Type II error

Type I: rejecting a null hypothesis that is true (a false alarm).

Type II: failing to reject a null hypothesis that is false (a missed fire).

2. Describing the errors in context

A city tests its water. p = the proportion of homes with unsafe lead levels. $H_0 : p = 0.10$, $H_a : p > 0.10$. If H_0 is rejected, the city replaces the pipes.

What the exam wants: the error and its consequence

Type I. The city concludes that more than 10% of homes have unsafe lead when really only 10% do. *Consequence:* money is spent replacing pipes that were fine.

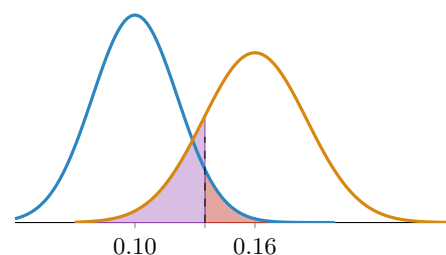
Type II. The city does not conclude that more than 10% of homes have unsafe lead when really more than 10% do. *Consequence:* people keep drinking unsafe water.

Here the Type II error is more serious, so the city should use a *larger* α (0.10 rather than 0.01).

3. How likely is each error?

- $P(\text{Type I}) = \alpha$. You choose it.
- $P(\text{Type II}) = \beta$ depends on the true value.
- **Power** = $1 - \beta$: the probability that the test correctly rejects H_0 when a specific alternative is true.

With $n = 200$ and $\alpha = 0.05$, H_0 is rejected when $\hat{p} > 0.135$. If really $p = 0.16$: $\beta \approx 0.17$ and power ≈ 0.83 .



4. The trade-off, and how to gain power

With everything else fixed, **lowering** α **raises** β : at $\alpha = 0.01$ the cutoff moves to 0.149 and β grows from 0.17 to 0.34.

Power increases with

1. a **larger sample size** (with $n = 400$ the power rises from 0.83 to 0.97, and α stays 0.05);

2. a **larger** α (at the cost of more Type I errors);
3. a true value **farther from the null value**.

Where students lose points

1. Swapping Type I and Type II.
2. Describing an error with no context.
3. “Accept H_0 ” in the description of a Type II error.
4. Thinking a smaller α makes the test better in every way.
5. Confusing power with the p-value or with α .

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

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