

Difference of Two Means

AP Statistics | Unit 7 | the two-sample t interval and the two-sample t test

Running example. Commute time in minutes, two independent random samples.

	n	mean	SD
City A	40	32.5	8.0
City B	35	28.1	7.2

1. The standard error of a difference

The parameter is $\mu_A - \mu_B$, the difference between the two *true* means; the estimate is $\bar{x}_A - \bar{x}_B = 4.4$. For independent samples, **variances add**:

Standard error of $\bar{x}_A - \bar{x}_B$

$$SE = \sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}$$

$SE = \sqrt{\frac{8.0^2}{40} + \frac{7.2^2}{35}} = \sqrt{1.600 + 1.481} \approx 1.755$ minutes. (Not $\frac{s_A}{\sqrt{n_A}} + \frac{s_B}{\sqrt{n_B}} = 2.48$: add variances, not standard deviations.)

2. The two-sample t interval

$$(\bar{x}_A - \bar{x}_B) \pm t^* \sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}$$

Degrees of freedom come from the calculator (2-SampTInt): here $df \approx 72.9$, so $t^* \approx 1.993$. By hand, the safe choice is the smaller n minus 1.

$$4.4 \pm 1.993(1.755) = 4.4 \pm 3.50 \Rightarrow (0.90, 7.90).$$

We are 95% confident that the true mean commute in city A is between 0.90 and 7.90 minutes longer than the true mean commute in city B. Always give the direction.

Because 0 is not in the interval, “no difference” is not plausible: there is convincing evidence that the two true means differ.

3. The two-sample t test

$H_0: \mu_A - \mu_B = 0$, $H_a: \mu_A - \mu_B \neq 0$, with both parameters defined in context.

$$t = \frac{(\bar{x}_A - \bar{x}_B) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} = \frac{4.4}{1.755} \approx 2.51, \quad df \approx 72.9, \quad \text{p-value} = 2P(t \geq 2.51) \approx 0.014.$$

Because $0.014 < 0.05$, we reject H_0 : there is convincing evidence that the true mean commute times in the two cities differ.

4. The conditions, for both samples

- **Random:** two *independent* random samples, or random assignment to two groups.
- **10%:** each sample is at most 10% of its population (not needed for experiments).
- **Normal / Large Sample,** for *each* sample: $n \geq 30$, or a graph with no strong skewness and no outliers. Here $40 \geq 30$ and $35 \geq 30$.

If the same individuals are measured twice, or subjects are matched in pairs, the data are *not* two independent samples: use a matched pairs procedure instead.

Where students lose points

1. Adding standard deviations instead of variances in the SE .
2. Not defining both parameters, or losing track of the order $A - B$.
3. Checking the conditions for only one sample.
4. Using two-sample t on paired data.
5. An interpretation with no direction: who is larger?

Now practise it

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