

Comparing Two Proportions

AP Statistics | Unit 6 | interval and test for a difference of proportions

Running example. 2000 website visitors are randomly assigned to one of two sign-up buttons. Button A: 120 of 1000 click ($\hat{p}_A = 0.12$). Button B: 150 of 1000 click ($\hat{p}_B = 0.15$).

1. The parameter is a difference

$p_B - p_A$ = the difference between the true click rates of the two buttons; the estimate is $\hat{p}_B - \hat{p}_A = 0.03$. Define *both* proportions in words and keep the order of subtraction fixed throughout.

2. A confidence interval for the difference

Two-sample z interval for a difference of proportions

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$$

Conditions. *Random:* two independent random samples, or random assignment in an experiment (as here). *10%:* for each sample when sampling without replacement (not needed for a randomized experiment). *Large Counts in both groups:* 120, 880, 150, 850 are all at least 10.

$$SE = \sqrt{\frac{(0.12)(0.88)}{1000} + \frac{(0.15)(0.85)}{1000}} \approx 0.0153, \quad 0.03 \pm 1.96(0.0153) \Rightarrow (0.0001, 0.0599).$$

We are 95% confident that the interval from 0.0001 to 0.0599 captures the difference ($B - A$) in the true click rates of the two buttons.

For a difference the key value is **0** (no difference). Here 0 is not in the interval, so there is convincing evidence that the buttons differ (only just: the lower end almost touches 0).

3. The test: why we pool

$$H_0 : p_B - p_A = 0 \quad H_a : p_B - p_A > 0$$

If H_0 is true, both groups share one proportion, so combine the samples to estimate it:

Pooled (combined) proportion and the test statistic

$$\hat{p}_C = \frac{x_1 + x_2}{n_1 + n_2}, \quad z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}_C(1 - \hat{p}_C) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$\hat{p}_C = \frac{120 + 150}{2000} = 0.135, \quad z = \frac{0.03}{\sqrt{(0.135)(0.865) \left(\frac{1}{1000} + \frac{1}{1000} \right)}} \approx 1.96, \quad \text{p-value} \approx 0.025.$$

Large Counts for the test uses $\hat{p}_C$: $1000(0.135) = 135$ and $1000(0.865) = 865$.

Because the p -value of 0.025 is less than $\alpha = 0.05$, we reject H_0 . There is convincing evidence that the true click rate for button B is greater than for button A. Because visitors were randomly assigned, we may conclude that button B *causes* the higher click rate.

4. Interval or test?

Confidence interval	each sample's own $\hat{p}$ (unpooled)	how big is the difference?
Significance test	the pooled $\hat{p}_C$	is there a difference at all?

Where students lose points

1. Pooling in the interval, or not pooling in the test.
2. Checking Large Counts in only one group.
3. Switching the order of subtraction halfway.
4. Looking for 0.5 in the interval instead of 0.
5. Parameters not defined, or hypotheses about $\hat{p}$.

Now practise it

Theory sticks once you use it. On **sad-academy.com** you'll find practice questions for this exact subtopic in the AP format, and an AI that grades your free-response answers against the scoring guidelines, the way an AP reader would.

sad-academy.com

Data are hypothetical. AP[®] is a registered trademark of College Board, which is not affiliated with and does not endorse this material.